

Functions

What you should know

A function is a mathematical rule that gives one output for a given input.

$f(x)$ notation for functions: e.g. $f(x) = 2x + 4$ would give the output 10 for the input 3.

This can be written as $f(3) = 10$.

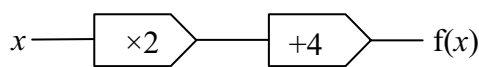
New idea

If you know the rule for the function that gives you the output from the input you can reverse this to find a rule that gives the input from the output.

This is known as the **inverse function** and is often written as $f^{-1}(x)$.

Task: Finding inverse functions

The function $f(x) = 2x + 4$ can be represented as:



You can then reverse this by doing the opposite operations from right to left.

So you can write the inverse function as



$$f^{-1}(x) = \frac{x-4}{2}.$$

You can check this using an output you already know, $f(3) = 10$. $f^{-1}(10) = \frac{10-4}{2} = 3$.

Find the inverse of each of these functions.

- 1 $f(x) = 3x - 5$ 2 $f(x) = 4x + 7$ 3 $f(x) = \frac{x}{2} + 1$ 4 $f(x) = \frac{x+2}{3}$
 5 $f(x) = \frac{2}{3}x + 3$ 6 $f(x) = 3 - 2x$ 7 $f(x) = x^2$ 8 $f(x) = \sin x$

In each case, check your answer using two or three outputs of the original function.

Take it further

- Find the inverse of some other functions. Some interesting ones are $f(x) = x$, $f(x) = 1 - x$, $f(x) = \frac{1}{x}$ and $f(x) = x^2 + 2x + 5$. (Hint: Try completing the square.)
- Using either pencil and paper or a graph-plotter draw the graphs of functions and their inverses on the same axes. (Use the same scale for both axes.) What do you notice?
- Find a function to convert miles to kilometres. Find the inverse function, which converts kilometres to miles.
Find a function to convert temperatures in Centigrade to temperatures in Fahrenheit. Find the inverse function.
You could try this with other conversions such as pounds to euros.
- Mathematicians define rules as functions if they have only one output for a given input. Why are the inverse functions for $f(x) = x^2$ and $f(x) = \sin x$ not strictly functions? How could you restrict them so they were functions?

Where this goes next

At A level inverse functions are studied in Core Mathematics.