MATHEMATICS TO KEY STAGE 4: MEETING THE NEEDS OF ALL ABILITIES

SEMINAR REPORT

Nuffield Curriculum Centre
and
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ABOUT THIS REPORT

This report covers a day in which some 40 people came together to consider the needs of young people in mathematics and how the planned new qualifications structures can best be designed to meet those needs.

Almost all of this report consists of the words of those at the seminar, either in the presentations or in the subsequent discussions at the various tables. Since everyone contributed whole-heartedly to the discussions, the number of authors is almost exactly the same as the number of people present. (See the list in Appendix B on page 26.)

As the day unfolded, the topics for discussion at the various tables became more varied, sometimes going on to new subjects and at others revisiting previous conversations. In compiling this report, I have set out to attach the various comments to the issues they refer to, rather than to the time of day when they were said.

Most of the comments that came from the rapporteurs are reproduced. The criteria for non-inclusion were repetition of essentially the same comments and (in just a few cases) being rather far away from the theme of the day. I have grouped the comments together under various headings and in some cases added a sentence or two of clarification.

I would like to thank everyone for their contributions to this report, and particularly the presenters and the rapporteurs (Richard Browne, Tom Button, Gerald Goodall, David Holland and Bernard Murphy) for their full accounts of what went on.

Roger Porkess

28/2/07

EXECUTIVE SUMMARY

The seminar looked at the needs of the spectrum of GCSE students, and how they might best be met when the new qualifications structure involving two GCSEs and Functional Mathematics is implemented.

The seminar was attended by 40 people, many of them senior figures in the world of mathematics education. (See the list in Appendix B on page 26.)

The day was much appreciated by those present and many favourable comments were received.

There were 4 presentations, covering:
- the needs of weaker students
- those of stronger students
- creative approaches to assessment
- a proposed overall structure.

Considerable time was allowed for discussion with people seated at five tables.

It was proposed that a pilot should be conducted, with GCSE 1 and Functional Mathematics based on a Realistic Mathematics Education (RME) curriculum, and GCSE 2 on more formal mathematics.

In the RME approach, students work through contexts which allow them to develop their understanding of mathematics. It is based on very substantial research. There is evidence that almost all students benefit from this approach, with particularly strong gains among the less able, who stay connected with the subject.

RME is used in the Netherlands where it has been developed at the Freudenthal Institute. It is also used in parts of the United States with leadership from the University of Wisconsin. Manchester Metropolitan University, with support from the Gatsby Charitable Foundation, are currently running trials of its use in England.

Serious consideration was given to the needs of more able students and how they might be met; many of those present were from Higher Education and they were broadly supportive of the proposed structure.

It was recognised that a new curriculum will only improve the situation if it is supported by appropriate assessment and another strand of the day was the creative assessment methods used in the Free Standing Mathematics Qualifications.

A recurrent theme in discussions was the need for relevant CPD for all teachers.

There was general agreement that a pilot along the lines proposed would be in the national interest.

INTRODUCTION

This seminar was a joint initiative from the Nuffield Curriculum Centre and Mathematics in Education and Industry (MEI).

It was set up at a time of very considerable national dissatisfaction about pre-16 mathematics, and particularly about GCSE.

- Employers report that those entering the workplace having studied mathematics up to GCSE are not functional in the subject.

- The number of students going on to take AS and A Levels in mathematics is insufficient to support national requirements, both for people proceeding to degrees in Science, Technology Engineering and Mathematics subjects and also for others whose careers will demand mathematics beyond GCSE.

At the time of the seminar two major changes were planned: the introduction of a qualification in Functional Mathematics and the move to two GCSE mathematics qualifications. These provided an opportunity to do much to improve this situation but this would only happen if the changes were soundly based. This seminar was born out of a concern that this should indeed be the case.

There were four sessions. The first two placed particular emphasis on the needs of certain groups of students and how they can be better met, given appropriate teaching and curriculum. This could well raise the need for more diverse assessment methods than those provided by the conventional type of examinations alone; this was the subject of the third talk. The final session looked at how a proposed new outline structure could be used to meet the needs of everyone.

The invitation to the seminar included the following words.

We believe that all the elements necessary for a substantial improvement in the understanding of mathematics acquired by our young people already exist. However, they need to be brought together into a coherent programme, involving appropriate styles of teaching and learning, associated curriculum and resource development, and considerable professional support for teachers. All of this then needs to be backed up and supported by the qualifications and assessment structure.

The purpose of this seminar is to develop a much clearer understanding of the new opportunities and of what steps need to be taken to turn them into reality.

PRESENTATION 1
REALISTIC MATHEMATICS EDUCATION: FUNCTIONALITY AND MATHEMATICAL DEVELOPMENT

Frank Eade, Manchester Metropolitan University
Sue Hough, Manchester Metropolitan University

The focus of the presentation was to consider the mathematical needs and achievements of lower and middle attaining pupils at KS4. Sue took a critical stance towards our current approaches to problem solving and functionality. The strategy of taking a real problem and turning it into a mathematical model was explored and it was suggested that, while this may be helpful to students in solving 'application' problems towards the end of a taught topic, it is also a complex strategy that many struggle to make sense of. Hence, this is not a notion which will support curriculum design or help in the development of learning trajectories.

Sue then provided examples from a recent questionnaire given to lower and middle ability Year 10 pupils which demonstrated both a lack of progress in terms of functionality and also pupils' poor grasp of the fundamentals of mathematics. Only 25% of the sample was able to work out the answer to 1/4 + 1/2 correctly. Of greater concern, however, was the fact that so few of these pupils had any conviction as to whether they were right or not. Interestingly, only those who had drawn a diagram or related the problem to a context had any confidence in their solutions. This led Sue to question how pupils gain authority over their mathematics and suggested that, without this authority, they cannot be considered to be 'functional mathematicians'.

Sue used a textbook called Some of the Parts to exemplify an alternative approach to teaching fractions and proportional reasoning. Sue emphasised that the whole book is concerned with contexts, and it is through engaging with these contexts that pupils come to make sense of the mathematics. Pupils are expected to develop increasingly sophisticated strategies and, rather than moving too quickly to formal mathematics, they gradually impose structure on their own ideas through the use of models and sense making.

From experience of a KS3 project using Realistic Mathematics Education (RME), funded by the Gatsby Charitable Foundation, Frank outlined a model of pupil development which took account of both functionality and mathematical development. The hypothesis is that for pupils to become effective mathematical modellers there is a need for curriculum designers to use reality as a source of mathematics and as a way through the mathematics. Unless the mathematics makes sense to pupils they have little hope of becoming functional. It is vital that pupils are supported in developing more sophisticated ways of thinking as they develop mathematically. Some examples of the different achievement profiles of project and control pupils were examined based on two years of trials. As well as having higher achievement levels it was also clear that project pupils made much

greater use of structured drawings and related purely mathematical problems to their own experiences.

Although the focus of the presentation was on the needs of lower and middle attaining pupils the outcomes for the very highest achieving pupils were at least as good as those for matched control pupils. It should be emphasised that one of the founding principles of RME is that mathematics should begin in a domain where pupils can make their own sense of it. As pupils develop, this domain does not have to stay in real contexts but can become the world of formal mathematics provided that pupils are secure with the subject at that level.

Frank emphasised that although this approach to curriculum design and teaching was new from the UK perspective it was based on 35 years of research in the Netherlands and about 16 years adapting it to needs of pupils in the USA. What was clear from the experiences in both the USA and the Netherlands was that there was a need to have an appropriate serial CPD package linked to the teaching approach. This package must deal with the beliefs, knowledge and the practical aspects of this approach to teaching. The evidence from the current project was that teachers can start to work with RME quickly and, with continuing support, become very effective indeed.

Evidence from the USA and the Netherlands indicates that teachers are more likely to embrace effective practices if the assessment system is supportive of this way of working.

Discussion relating to Presentation 1

While some of those present were familiar with RME this was by no means true of everyone. This was reflected in quite a lot of the conversation at the tables and at times a measure of hesitancy. However, the overall feeling was that this is a promising development that is worth pursuing.

Those present liked the textbook material they had seen in the presentation but were aware that it amounted to only a few examples and this left several people wanting to see more and to know what it is like teaching with such material for more than a few lessons.

The presentation concentrated on the use of RME with weaker students and several people, at different points of the day, asked whether there are materials for its use with stronger students. The answer is Yes, and that it has proved successful with such students.

There was general awareness that a very considerable programme of CPD would be needed if RME were to be introduced into this country.

Many questions were asked about the research underpinning RME and Appendix C outlines the sources of information about this.

The RME approach

Some of the group knew little (or nothing) about RME before this session. There was a consensus opinion that this sounded as if it might work, was worth piloting and it seemed an improvement on what happens at present to low achieving children.

The group were very positive about this approach but if teachers are to receive proper training and to develop their practice then there will inevitably be a time when they are finding their way and consequently are not teaching as effectively as they might. How do we give teachers the space to 'fail' whilst learning? Exam league tables will not go away in the meantime.

I would like to see promotion of the teaching approaches of Mathematics in Context more widely throughout the teaching profession, including teachers of numeracy for adults.

Curriculum and pedagogy

Do we assume that pupils can do less than they actually can, i.e. invent ceilings that aren't there?

This seems like a very good way to teach Functional Maths and GCSE 1 but questions were raised as to its adaptability to GCSE 2. Perhaps there comes a time when the basics are so thoroughly understood that students should be expected to cope with more abstract, out of context ideas. For algebra, number is the context, i.e. the context can be mathematical itself.

Why do we ask primary children to formalise their mathematics so early? They are required to do 'sums' before they understand what they mean. Let them be practical for as long as they need.

Mathematising is crucial, and is given proper weight in RME. The Functional Mathematics Standards also give full weight to mathematising, in the 'representing' section.

Assessment

Without sympathetic assessment, RME will simply seem to be a waste of too much valuable time.

A critical issue for all abilities is to emphasise the question 'How do you know your solution is right?'

Materials

The choice of material and its development seems to be a sophisticated process based on research. How robust is the system? How does a teacher leave some out or produce further material?

Is the textbook full of good ideas but too much like a script that must be plodded through linearly? The format of any materials must be carefully thought through so that they are not a recipe but encourage real engagement.

We recognised that some bright students are able to work abstractly very rapidly; we could see the value of the material we had been shown, and the RME

approach, in helping other students to retain the ability to make sense of mathematical questions, by rooting these in concepts that are securely understood.

The RME approach needs to be customised for England (and Wales?) both in content and to use the best of our current practices.

CPD

RME forces teachers to think about concepts underlying mathematics and the ways they are acquired by young people. The latter needs to be a major focus of CPD.

When teaching RME, it is important for teachers to unlearn their formal approaches.

There is a need for loads of CPD to change teachers' beliefs about how to approach mathematics.

I would like to see the use of the National Centre to set up cluster groups and networks, or to use existing networks, to promote these approaches.

PRESENTATION 2 HIGH CHALLENGE, HIGH INTEREST

Dr Jennifer Piggott, Director of the NRICH Mathematics Project
Stella Dudzic, Programme Leader (Curriculum), MEI

Jennifer asked colleagues to share the experience of working on some mathematics together; questions were taken from these sources:

- Edexcel 1387 Mock Paper 5,
- NRICH ('Squares'),
- Edexcel 1387 June 2005 Paper 5,
- NRICH ('Pair products').

Stella followed up with a brief presentation which is outlined below.

In common with everyone else, the most able students need a curriculum that will interest them, provide an appropriate level of challenge and equip them with skills which they are able to use. They also need a firm foundation of knowledge and understanding for possible future study - this could be A-level mathematics and beyond or it could be some other numerate subject.

The four questions which colleagues worked on at the beginning of the session illustrated these points.

- The same content can be used in very different contexts - able students should be encouraged to realise this and to see the connections between different areas of mathematics.

- Examination questions have an influence on teaching and learning, particularly via published text books.

- The use of investigational questions in assessment did not lead to all students engaging in thinking for themselves and making connections in mathematics.

- What is interesting and motivating for one student may not be equally interesting and motivating for another.

- The relationship between assessment and teaching and learning is complex.

Currently the mathematical experiences of our students include:

- Social views of the subject - it is acceptable to say you are no good at mathematics

- Emotional experiences of the subject - fear of failure of students and their parents at home and in the workplace

Negative feelings about mathematics are not confined to people who are not successful at school.

Acceleration and early entry for GCSE is the most commonly used strategy to meet the needs of the most able. Ofsted's Evaluation of Mathematics Provision for Ages 14-19 (May 2006) found that:

Early entry can be successful in circumstances where the students all have the ability to achieve high grades and there is a carefully planned learning pathway available which ensures an appropriate depth of understanding of mathematics beyond GCSE ... Inspection evidence suggests, however, that only a small minority of schools are in a position to use such a policy effectively.

There is no evidence that students who have taken early entry do better in maths related subjects in HE. Does taking GCSE Mathematics early translate to increased engagement with the subject?

To improve learners' experiences we need to do the following.

- Rethink the structure of the mathematics experience in schools including the value of acceleration and taking GCSE early.
- Challenge students' preconceptions about their ability to study mathematics and about the nature of the subject. Being 'stuck' can be part of the learning process.
- See entitlement as more than curriculum coverage.
- Improve teachers' confidence so that they are able to take control of the learning experience rather than just working through the text book.
- Acknowledge that there may be more than one way to meet targets and that concentration on meeting targets can have negative educational effects as well as positive ones.
- Make use of Functional Maths and changes to the National Curriculum at Key Stage 3 and 4.
- Assess process as well as skills.

Questions to consider:

- Do students' experiences of mathematics leave them wanting and able to do more? How can a positive experience be encouraged?
- How should the new GCSEs in mathematics be structured to improve the learning experience?

Discussion relating to Presentation 2

This presentation promoted diverse discussion. Much of it related to problems that prevent stronger students from fulfilling themselves, and so may be relevant to the values and culture of new GCSE qualifications, and particularly GCSE 2. Topics included: what makes mathematics interesting; acceleration; the RME approach; CPD and textbooks

There were strong feelings about the poor management of the education of many strong mathematicians.

What makes mathematics interesting?

Highly challenging problems should not be restricted to the most able; all students should be exposed to interesting and challenging mathematics and be aware that pleasure can be gained from it.

- *A lot of people do not like number puzzles.*
- *A lot of people like mathematical puzzles, e.g. Sudoku.*

It is important that such problems are accessible, even if they are difficult. There should be a way in for less able students.

The reason given why a piece of mathematics is useful has to be relevant to learners at the time of learning. It is not satisfactory to say simply 'This will be important later'.

Sometimes it would help to introduce vignettes of historical information, e.g. what problems were mathematicians facing when they developed quadratic equations ? This would potentially motivate more effective learning, not just enrich the curriculum.

Acceleration

Acceleration tends to put students off mathematics. Bright students need a different approach from others, not just to be pushed to cover the curriculum more quickly. Frank Eade had questioned bright students and they are frequently disillusioned by the subject. Top sets can be taught in a particularly pressured way.

We should recognise that there is something special about mathematics. Only in mathematics and music (and perhaps sport) is rapid acceleration even feasible. What is it that makes this possible?

Assessment

Several teachers feel that sometimes too much emphasis is placed on having to do mathematical problems quickly. Why is it necessary to have timed exams?

The RME approach

Most of the discussion centred on the use of RME for more able children. It was recognised that it would be a bonus if this were possible. It was hoped that the RME approach transferred to more formal work in mathematics. The view was expressed that in the Netherlands, university mathematicians felt that RME was not suitable right up to university entrance level. Later in the day it was made clear that the RME approach had been developed for more formal mathematics; the brief for Session 1 had been to discuss the suitability of RME for the less able. Also, the experience of universities in the Netherlands could be dependent on other factors.

Textbooks

Design of textbooks and other materials should take account of how they will actually be used in the classroom.

Appropriate textbooks are very supportive (and this includes encouragement to look to other sources). Textbooks often include enrichment material, but it can be bypassed by the teacher.

There is a responsibility on authors to develop resources appropriately.

A major issue is that textbooks are too closely linked to examinations. It is beginning to happen that parents insist on the "book for the exam" being used.

There is too much (and increasing) pressure on teachers from too many initiatives, of which too many are ill-considered, and this leads to a perceived need for a quick fix, i.e. very directed textbooks.

CPD

Teachers should be coming into the profession with an expectation of CPD and a responsibility to engage in it. There may not be enough time in PGCE courses, but this wouldn't matter (so much, anyway) if there was good CPD. Management has a responsibility to enable CPD, especially subject specific.

The aim must be to bring about an approach that is not strictly teaching to the exam but more broadly. All pupils, including the weaker ones, will end up doing better and there is evidence to that effect which must be promulgated.

PRESENTATION 3 CONNECTING MATHS WITH STUDENTS AND STUDENTS WITH MATHS

Geoff Wake, Lecturer in Mathematics Education, University of Manchester

Geoff Wake asked the question *"In developing Functional Mathematics, what might we learn from the 'innovative' forms of assessment of Free-standing Mathematics Qualifications (FSMQs)?"* In doing so he suggested that the development of Functional Mathematics might provide the opportunity to (re-)connect mathematics with students. He proposed that there is clear evidence from the reduction in numbers of those electing to study maths post-16 over recent years that the subject does not always have appeal to students who increasingly elect to study subjects that perhaps have more easily adapted to reflect the interests of young people in the 21st century. Geoff suggested that perhaps there are a number of lessons that might be learned from the development of FSMQs that might usefully inform the development of Functional Mathematics as these qualifications had to address some of the very issues that are pertinent to the case of Functional Mathematics.

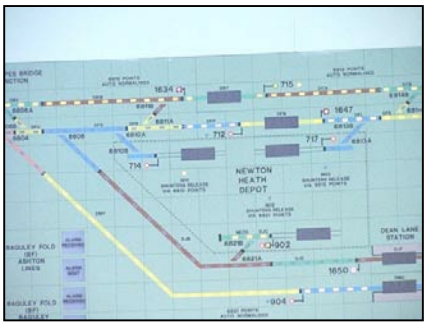
In considering how innovative practices in the assessment of FSMQs might usefully inform the assessment of Functional Mathematics, Geoff started by suggesting that the design and development of a curriculum specification and its assessment need to be informed at every stage by a clear vision of what the designers want to be happening in classrooms in terms of the mathematical activity of learners.

Geoff went on to reflect on how, and why, particular assessment practices of FSMQs, such as the use of portfolios and of data sheets in timed written exams were developed. It was, he recounted, an aim in developing FSMQs to influence what happens in classrooms in terms of teacher practice and ultimately learner experience as it was the intention to provide courses at different levels that would prove attractive to those students in the post-compulsory sector who would usually give up study of mathematics at age 16. The intention, therefore, was to develop a course that organised mathematics in such a way that its immediate or potential use was apparent to students and so that the process of learning and doing mathematics more realistically reflected how this is possible in a technology rich world. At the core of the design of curriculum and assessment process there was the desire that students should be involved throughout the course with substantial and meaningful mathematical activity with modelling and applications at its core and that wherever possible this would involve technology both as a tool for learning and as a tool for doing mathematics.

In designing FSMQs, Geoff had turned to previous curriculum development and research with which he had been involved in Manchester which suggests that to apply mathematics successfully in solving meaningful problems students need to bring together a coherent body of mathematical knowledge, skills and understanding to undertake substantial tasks. This moves one away from thinking of the mathematics curriculum as being a collection of atomised mathematical

skills (often the result of other curricula and assessment), towards consideration of common ways in which mathematics can be harnessed to analyse situations and solve problems.

To illustrate what functionality with mathematics might possibly entail, Geoff very briefly drew on his research into applications of mathematics in workplaces and classrooms. As the photographs and workplace tables and charts he used from one case study based on railway signalling immediately demonstrated, mathematical activity in workplaces appears very different in nature to that in school classrooms and almost always incorporates the use of technology in some way. Workers are often involved in mathematical activity in workplaces, Geoff suggested, that involves making sense of mathematics that has been developed by others.



Students investigating workplace activity in a railway signal box.

Geoff went on to propose that the first arena in which we might expect students to require functionality with mathematics would be in classrooms of other school subjects. He hoped that Functional Mathematics might allow students to be more successful with this than is currently the case. He described some findings from a small-scale research project that highlighted students' lack of functionality with mathematics in science classrooms. The students in an AS Physics classroom would ordinarily be considered able mathematicians studying both AS Maths and AS Further Maths. However they appeared very unsure about what to do mathematically when asked to consider whether data concerning two variables they had collected in an experiment were directly proportional. Follow-up activities confirmed a lack of deep understanding of the concept and when asked to interpret mathematical representations in terms of the real situation the students were unable to do so.

The only way to ensure that students are given opportunities to work with mathematics in a functional way, Geoff proposed, is to ensure that the assessment process promotes this. In the development of FSMQs it was believed that timed written examinations are not suitable in this regard: hence the use of portfolio assessment. In an attempt to make an impact on classroom experience students are expected to develop and build their portfolio over the duration of the course and the best quality work is expected to be seen in later pieces of work within the portfolio. The development of the portfolio becomes the motivation for learning: thus the assessment really does focus on course work. The timed written examinations attempt to mirror the demands of the portfolio: the pre-release data sheets allow students to become familiar with a context that is to be examined and during the examination they work on this in much greater detail than would

usually be felt to be advisable. Additionally at AS Level students have a mathematical comprehension examination in which they make sense of the mathematics of others allowing them to develop some of the skills they might require when applying mathematics in other situations.

In conclusion Geoff asked delegates to consider two questions:

- What do we want students to be doing in terms of functional maths?
- What assessment is fit for this purpose?

Discussion relating to Presentation 3

The discussion following this presentation was largely based on the themes of curriculum, pedagogy and assessment, and the relationship between them.

Curriculum

What do we want students to be doing in terms of functional mathematics?
• To be able to read mathematics and use ICT.
• To have the mindset that 'I can work it out',
• To see links between mathematics and the common sense approach of their unconventional methods.

Functional Maths should consist of tackling new or non-routine problems from everyday life, using commonsense as well as mathematics to solve them.

Above all else, we need to teach a conviction that mathematics can be used to solve practical problems. Together with confidence to use mathematics, this is much more important than covering a large curriculum.

We need to work on helping learners to realise how mathematical the world is; in particular teachers need to have thought more about this point. The contexts chosen for these lessons need careful thought; in many instances the mathematics behind an everyday situation may be too complex, e.g. the flight-deck of a jumbo jet.

It is hard to get learners of school age to engage with 'adult' problems; we need to set simple tasks where noise doesn't obscure the key message. We need to explore what problems are most effective.

Proportional reasoning is absolutely essential as a basis for understanding mathematics and its use in other subjects and the real world.

Pedagogy

Bring in team working. It encourages discussion, encourages familiarisation with the problem, prepares for the real working environment, probably assessed by group projects and assessment of individual contributions within the group. Management of this for large groups needs to be looked at in the pilot. There needs to be flexibility and creativity in managing classes. There also needs to be good design of tasks and reporting mechanisms.

Overall, what is needed is not teachers that can give impressive justifications of the importance of mathematics, but rather teachers whose practice drip-feeds the message through continuous use of examples that suggest this.

What is needed is to change classroom experiences and teaching methods; it will also be essential to change the way teachers are trained.

Assessment

The group was enthusiastic about the use of some of the FSMQ assessment techniques, at least for a GCSE 1 with the RME approach. The following was thought to be highly suitable.
• Written examinations should have some materials sent to the centres for the candidates to become familiar with a context before the examination.
• We should encourage the use of questions that started in context and then became more general.
• We should encourage assessment that requires more communication from the candidates than simply writing down numbers.

What assessment is fit for this purpose? The model used in FSMQs but extensive trialling is essential; timed exams do not allow for try-and-try-again which is what being functional in mathematics is all about.

At the moment, assessment drives the syllabus rather than the other way round. Innovative forms of assessment are needed which encourage good practice.

There is a 2-way link between assessment and teaching/learning. Each one affects the other, often in a negative way which leads to the lowest common denominator. For improvements to be made there need to be brave changes in both.

There was much enthusiasm for pre-released information sheets, and not just data (i.e. purely numbers) sheets. Assessment must be unseen but can be based creatively on these sheets. We encourage discussion, use of the internet, teaching around the pre-released sheets, etc. We note that pre-release sheets overcome many administrative problems, e.g. language, DDA.

Assessment needs to be holistic (i.e. addressing the whole problem, not atomistically ticking off techniques).

Even if problem-solving type questions are introduced into assessment the impact and value of these can be lost as teachers become familiar with the style of them and "teach to the test". The obvious example of this is GCSE coursework.

A lot of GCSE 1 and Functional Maths should be teacher assessed.
However, this could be very difficult if the results count towards league-tables and teachers' pay. Likewise, oral assessment would be easier if it weren't grade-dependent.

Assessing via a portfolio, as it is done at FSMQ, can work.

Oral assessment can be useful, but this needs to fit in with the whole assessment process.

PRESENTATION 4
A STRUCTURE TO MEET THE NEEDS OF ALL

Dr Tony Gardiner, Department of Mathematics, University of Birmingham
Roger Porkess, Chief Executive, MEI

This presentation focused on the need for a coherent qualifications structure, taking account of the points made in the previous sessions. Three different qualifications are planned for 16-year olds: Functional Mathematics, GCSE 1 and GCSE 2. The proposal put forward is that GCSE 1 should be based on RME with Functional Mathematics embedded in it. GCSE 2 should involve more formal mathematics.

Tony spoke first, providing a view from Higher Education. He offered two examples showing aspects of mathematics and issues that a successful design of GCSE 2 needs to address to meet the needs of future undergraduates in numerate disciplines: the ability to answer multi-step exercises reliably, and the ability to think about two things at the same time.

His first example, about two cyclists, involved basic arithmetic, but it was given in words and required students to perform four calculations. Although this is the sort of work one might associate with Year 11, evidence from several years shows that on a given day 25-30% of students with a grade A at A level, who have chosen to study for a mathematics degree, will fail to get the right answer. It seems that they have little experience of having to string together simple steps to solve multi-step exercises.

His second example involved calculating the angle between the hands of a clock at a specified time, or finding a time when the angle was of a specified size. Again students had to coordinate two things (the movement of the minute hand and the hour hand); but whereas with the cyclists the parts had to be handled consecutively, here the two aspects have to be handled simultaneously. Trial results were again disturbing, but indicated clear differences between serious mathematicians (e.g. those taking Further Mathematics, who recognised that there was something that one had to think carefully about) and those preparing to study engineering who could handle problems in which just one hand moves, but when required to coordinate the two hands simply charged ahead getting every single problem wrong.

Roger then took over and talked about the proposed structure. Would it work? Certainly all the key elements are in place for it to succeed.

• The decision to have Functional Mathematics and two GCSE qualifications has been made. It is now up to everyone to make sure they work well. QCA have drawn up standards for Functional Mathematics and have handed them over to a large number of awarding bodies, leaving them to produce workable schemes. This approach is doomed to failure. Students' lack of functionality cannot be solved by assessment alone; it requires suitable teaching to a suitable curriculum. If the teaching remains much as it is, students will learn (or fail to learn) much the

same things, and the assessment will soon adapt to measuring what they know. No awarding body will set papers which they know candidates are in no position to answer.

• Functional Mathematics will inevitably be taught within GCSE 1 and so functionality needs to be central to its curriculum and not an add-on. This could be achieved using RME. Evidence from the pilot being carried out in Manchester suggests that those students who follow such a course do indeed become more functional.

• The changes to the National Curriculum in the current review emphasise processes without losing important content. Basing GCSE 1 on RME is consistent with this; indeed it would allow these changes to take effect.

• Basing GCSE 1 on RME would build not only on the 3 years experience at MMU described in the first presentation, but also on the solid foundation of over 30 years research at the Freudenthal Institute.

• The assessment of GCSE 1 (incorporating Functional Mathematics) could not be based entirely on conventional examinations. However, suitable alternatives exist and have been used for other qualifications in recent years, particularly FSMQs, as described in the previous presentation.

• Most teachers have no knowledge of RME and so an extensive programme of professional development would be needed. The NCETM is now in place and is the ideal organisation to manage such provision.

• GCSE mathematics is no longer to include coursework and this leaves a vacuum. Coursework was designed to ensure students encountered processes that are central to using mathematics. No one knows how such work is now going to be assessed, or indeed whether it will take place at all. However, basing GCSE 1 on RME could overcome this problem.

Roger then went on to consider the outcomes of adopting such a structure.

At the moment a large proportion of our young people come out of GCSE not only knowing no useful mathematics but actually immunised against it.

That situation would be changed. Furthermore, with good design (especially of GCSE 2), more students would continue mathematics after GCSE.

He then touched on the relationship between content and functionality. In the present curriculum content leads and functionality lags some years behind. However, advocates of RME would claim that with such an approach the two would be much closer together, with students learning content through engaging with realistic contexts. Research in the United States suggests that students who have a background of RME are more successful when making the transition to more formal mathematics.

The presentation ended by considering the consequences of doing nothing. The forthcoming changes open a rare window of opportunity and it would be a national tragedy were it not to be used. That is, however, the direction in which we are currently heading, with the prospect of change but no improvement.

By contrast, the proposed structure could greatly improve the current provision, but it will only achieve this if it is first subjected to the rigours of serious development work and a well-conducted pilot.

Discussion relating to Presentation 4

There was some overlap between earlier discussions and those following this presentation and some topics were re-visited at most tables.

At all tables the discussion on this presentation spread into the final session and relevant comments are recorded here.

The overall structure

The group gave general assent to the following proposal.
- *Functional Mathematics should be incorporated in GCSE 1 and should be awarded on the achievement of at least a certain grade at GCSE 1.*
- *GCSE 1 (at least) should be taught using the RME approach with some of the assessment using ideas from FSMQ, as outlined above.*
- *GCSE 2 should contain mathematics that is more formal than that found in GCSE 1.*
- *There is need for a pilot scheme.*

It was felt that the model being discussed was workable.

We think that a project of this nature (and there is no doubt potential scope for others) could "infiltrate" the existing pilot work.

Existing development of GCSE 1 and GCSE 2 is going ahead by the awarding bodies as an assessment-led process, whereas it is the delivery that matters; assessment should follow delivery and not vice versa.

Functional Mathematics

How/where will Functional Maths be taught/assessed? This needs to be decided very early in the planning stage.
- *As part of GCSE 1 or separate to it ?*
- *Will it be a hurdle to a 'pass', or have its score aggregated in the final grade?*

Functionality works in both directions – students need to be able to move from the real to the abstract and the abstract to the real.

For Higher Tier students, Functional Maths could appear as an unnecessary hoop to jump through.

Functionality does not only apply to Maths – it sits within the wider concept of functional skills.

GCSE 1 and 2

The fear is that many of the grade C students will be entered for GCSE 1 only. Entitlement to GCSE 2 needs to be enforced.

We discussed the extent to which GCSE 2 was an entitlement for all students, but did not agree about this. Some people thought that GCSE 2 was intended as a more challenging examination for the brightest, but a more widely held view was that the more students who took GCSE 2 the better. This led to the conclusion that there is, as yet, no clear consensus on the purpose for and design of the two GCSEs. We noted that there are at least four models currently (the one proposed today, the Kings/Edexcel model, the MA model, and the QCA model). If the two GCSEs are distinguished by different assessment approaches or expectations of formality, we need more and clearer examples to illustrate these.

The system needs to allow a late decision to be made by a student about whether GCSE 2 would be taken. The implications of the timing of this decision on time allocation, time-tabling and teaching approaches (e.g. RME) need to be considered.

Some thought that alternative ways of solving the same problems could distinguish the two GCSEs. Others were convinced that the two examinations would need to contain quite different styles of question, with GCSE 1 assessing grasp of basic procedures and GCSE 2 assessing ability to use these procedures to address more substantial problems.

We have some concern that GCSE 2 should not be entirely abstract. The GCSE 2 would need its own place in the pilot; and vice-versa that GCSE 1 is not entirely non-abstract.

RME specifies a teaching approach for GCSE rather than the content. It was not clear that the idea of an RME GCSE made sense. It would be better to investigate whether an RME teaching approach led to better results in a conventional GCSE. Similar distinctive teaching approaches, e.g. the Active Learning suggested by the DfES Standards Unit, could also be studied.

CPD

When mathematics departments look to improve results they almost always think in terms of booster classes and more time with the students; they do not generally think about teaching differently. This is essential and there are clear implications for extensive CPD.

The crucial question is: "How is it going to be taught in the classroom?"

Without substantial CPD, teachers faced with a new outcomes-based description of the curriculum would carry on teaching as they do now, and would do last-minute examination preparation that covered questions of the new type. For 21st century science, a 5-year programme was needed to embed the practice that was wanted.

End-users

In the proposed discussion questions it says 'Students leave school equipped to use mathematics in everyday life and employment'. It was felt that this will always be unrealistic. Instead employers need to appreciate that the goal is for their employees to have a positive attitude towards mathematics and the ability to think mathematically. Universities also need to be aware of what schools are trying to

achieve; there is a tendency for them simply to say that their students aren't as good as they used to be.

We need a structure that better prepares students for HE.

We need to consider the difficulties likely to be experienced if changes at pre-university level required universities to change their expectations at entry.

Looking ahead

We cannot yet again make changes to curriculum and/or assessment without adequate piloting.

Resources, and convincing QCA there is a need for new pilot work of this kind, are an issue. But if it fell on these grounds, all the opportunity would be missed. Creative development, such as we in this room can do, would also be lost, to the detriment of several generations of students and teachers.

We need to address questions about how the initiative fits in with decisions already made, and the existing timescale. If a research programme goes ahead, it could not match the government's plan of introducing new GCSEs from 2010.

There should always be pilots on the go and only taken on when proved to be successful. A long term (5 years or longer) pilot is needed.

An extensive research project is needed to look at what lessons have been learned in other countries. Dissemination of the findings amongst teachers is needed.

FINAL GROUP DISCUSSION

Participants were given a set of six questions for the final discussion. However, by this stage they were all deeply engrossed in conversations at their own tables and so no written answers to the questions were forthcoming.

However, the rapporteurs at most tables made quite extensive notes summarising their discussion. Many of these are points of detail and most of these have been treated as unfinished business from earlier discussions and incorporated where appropriate.

In addition the following summary statements were made.

We think there is scope for a project of this kind (without ruling out the possibility of others), integrating the delivery with creative development of assessment, taking account of research evidence and with appropriate continuing professional development.

I spoke to everyone at my table after the end of the formal session. All thought the day worthwhile. All thought we had a proposal that would make things better. All had some concerns but retained a positive attitude and seemed ready to accept that their own special interest might have to give some ground (compared with what might be done) in order to allow better general progress.

It was suggested that "Students leave school equipped to use mathematics in everyday life and employment" could be taught in citizenship and "Students are equipped for further study in mathematics" could be taught in mathematics. This was not universally agreed with. Mathematics properly taught can interest all students. The abstract part of mathematics is important for those who want to progress.

An extensive pilot out of the glare of the public spotlight is needed. It would benefit from private funding so that, unlike DfES funded pilots which have to succeed, there is the possibility it could fail! Clusters of good practice spread outwards: to make significant change the teachers need to control and drive the process forwards. This has been seen with the Standards Unit materials.

We discussed the question whether the proposal should involve a new requirement for all students. If so, how would teachers be prepared and by whom? Similarly we discussed whether the intention was to replace all existing GCSEs with new, RME-based GCSE qualifications. We were clear that neither approach would gain consensus. [In reply to this comment it was pointed out that the proposal was for a pilot and not for an instant change, and that a pilot is able to adapt to the information and opportunities that it generates.] *The new style of teaching does not need to be RME. However, we agreed that significant change of practice was needed and that RME is a good exemplar of an approach that works.*

IN CONCLUSION

After the seminar, an elder statesman of the mathematics community sent in this commentary on the day. It is reproduced in full as a conclusion to the report.

Overall, the proposed project sketched to us seems worthy of support. It would seem to have a good chance of producing a workable package which would handle the divide between GCSE 1 and 2 in a rational way.

One could view GCSE 1 as the course aimed at those not expecting to make serious use of mathematics henceforth - in other words, this would be maths for real life and for most workplaces. On the other hand, GCSE 2 would be the course aimed at all those who might be proceeding further in mathematical studies. Thus this would be maths for the future mathematicians, scientists, engineers, economists, technicians etc (as well as for those who simply enjoy mathematics).

The consequence of this is that AS and A2 level Mathematics would be designed on the assumption of success in GCSE 2. Whilst this need not be stated as an absolute pre-requisite, it would need to be the norm, and those wishing to proceed from GCSE 1 to AS without GCSE 2 would need a bridging course - which would have to be, essentially, GCSE 2 itself!

There remain problems making this fit with current DfES pronouncements. The relationship described between Functional Maths and GCSE 1 doesn't seem to me quite what the Government has in mind - even if it is the rational answer! In particular, there is a demand that Functional Maths should be 'mastered' by those who are deemed to 'pass' it. Additionally, the DfES line remains that GCSE Maths 1 should be a suitable preparation for AS Maths - but they have accepted, and said, that doing GCSE Maths 2 as well would be a better preparation!

We need to look closely at the relationship between RME and transition to numerate HE disciplines in the Netherlands. It would appear that there have been difficulties here - which we would do well to learn from and avoid.

It is hard to see how the project discussed could proceed without charitable support. The exam boards, who are now charged with developing the qualifications, do not appear to have the brief to consider how it should be taught nor how the teachers should be prepared; nor are they likely to come up with proposals which 'bend' the government's requirements; nor are they able to simply decide the pace is too fast, and spend more time on pilots etc. On the other hand, those who were involved in this seminar's proposals seem to be considering the whole problem and to be well able to move this forward.

APPENDIX A PROGRAMME FOR THE DAY

10.00 Arrival and refreshments

10.30 Welcome and introductions

10.45 **Realistic Mathematics Education: functionality and mathematical development** *Frank Eade and Sue Hough*
Discussion in groups

11.30 **High challenge, high interest: increasing understanding and motivation**
Dr Jennifer Piggott and Stella Dudzic
Discussion in groups

12.15 **Connecting maths with students and students with maths** *Geoff Wake*
Discussion in groups

13.00 Lunch

13.45 **A structure to meet the needs of all** *Dr Tony Gardiner and Roger Porkess*
Discussion in groups

14.30 **Identifying priorities and formulating recommendations**
Group discussion

15.15 **Summarising and action points**

15.30 Meeting ends

APPENDIX B LIST OF ATTENDEES

Stephen Abbott, HMI
Joan Ashley, NANAMIC & NCETM
Dr Anthony Bainbridge, Fellow, Institute of Engineering and Technology & MEI Trustee
Ken Barley, Chair of Examiners (Mathematics), OCR
Richard Browne, MEI
Tom Button, MEI & The Further Mathematics Network
Jenita Chelva, ACME
Diane Cochrane, ACME
Graham Cumming, QCA
Cynthia Davies, DfES, General Qualifications & Assessment Unit
Ruby Dlay, The Clothworkers Foundation
Stella Dudzic, MEI
Frank Eade, Manchester Metropolitan University
Doug French, President, Mathematical Association
Dr Tony Gardiner, University of Birmingham
Gerald Goodall, The Royal Statistical Society & MEI Trustee
Angela Hall, Nuffield Curriculum Centre & Science Learning Centre, London
Liz Henning, NCETM
Katharine Henson, Downe House
Derek Huby, Secondary National Strategy
David Holland, MEI trustee
Sue Hough, Manchester Metropolitan University
Professor Celia Hoyles, Chief Adviser for Mathematics, DfES
Andrew Hunt, Nuffield Curriculum Centre
Jane Imrie, NCETM
Sue Johnston-Wilder, Chair, Association of Teachers of Mathematics
Vinay Kathotia, The Royal Institution
Dr David Martin, Hon Secretary, JMC & Chair, NANAMIC
Greg Midcalf, DfES
Bernard Murphy, MEI & Teaching Advanced Mathematics
Dr Jennifer Piggott, NRICH Project Director
Roger Porkess, Chief Executive, MEI
Dr Phil Ramsden, Imperial College
Sue Robinson, The Gatsby Charitable Foundation
Professor Chris Robson, London Mathematical Society
Teresa Smart, London Mathematics Centre
Dr Malcolm Swan, The Shell Centre
Anthony Tomei, The Nuffield Foundation
Geoff Wake, University of Manchester
Drs Monica Wijers, Freudenthal Institute for Science & Mathematics Education, Utrecht University, The Netherlands

APPENDIX C REALISTIC MATHEMATICS EDUCATION: INFORMATION SOURCES

There were many questions about RME and the research underpinning it.

The major research is an on-going programme, now some 35 years years old, undertaken by the Freudenthal Institute in the Netherlands. Their web-site has some articles in English and some relevant software.

http://www.fi.uu.nl/en/welcome.html

Research has also been carried out at the University of Wisconsin in the United States. A brief summary of the project in the States can be found on the following web-site.

http://www.wcer.wisc.edu/news/researchNews/mathsincontext.php

The presentation was based on research at Manchester Metropolitan University, funded by the Gatsby Charitable Foundation, involving the use of RME in schools in Manchester and elsewhere. The MMU web-site gives details about the project and some results.

http://www.partnership.mmu.ac.uk/cme/DMtC/Default.html

Research into the application of mathematics has also been carried out in Germany (SINUS programme). See the site below for an outline of its implementation.

http://www.bmbf.de/en/1254.php

Gabriele Kaiser and Katja Maas have written extensively about this and associated research. A chapter from a book is available on this site.

http://www.erzwiss.uni-hamburg.de/Personal/Gkaiser/pdf-publist/icmikaiser-maass.pdf

There is also a paper by Gabriele Kaiser 'The Mathematical Beliefs of Teachers about Applications and Modelling – results of an empirical study' which is available as part of the proceedings of PME 30 (2006), but at the moment it is not available electronically.

The following site has a variety of materials developed in the USA as part of the Reform Movement.

http://showmecenter.missouri.edu/showme/mic.shtml

The Mathematics in Context series of textbooks is published by Holt, Rinehart and Winston.

http://www.hrw.com/math/