Changes to KS3 & 4 Programmes of Study in Mathematics
MEI response

Introduction

The changes to the Programmes of Study to increase the emphasis on understanding and the interlinked nature of mathematics are welcome. This change in emphasis is in line with the findings of Ofsted in "Evaluating mathematics provision for 14–19-year-olds" (May 2006): "The best teaching gave a strong sense of the coherence of mathematical ideas; it focused on understanding mathematical concepts and developed critical thinking and reasoning."[1] Removing the pressure to get through a fixed body of content, whether students understand it or not, would reinforce this change in emphasis, but this pressure is at least as much a result of the testing process as it is of the Programmes of Study.

Encouraging schools to make links between different subjects and to adapt the National Curriculum to produce something tailored to their students is excellent in principle. However, the repackaging of the current curriculum into the proposed Programmes of Study does not necessarily make it easier to achieve this. Moreover, the revised structure of the Programmes of Study for Mathematics does not seem to provide a good fit to the nature of mathematics as a subject and obscures what needs to be taught; this will be particularly problematic for less experienced teachers.

Key Concepts

"There are a number of key concepts that underpin the study of mathematics. Pupils need to understand these concepts in order to deepen and broaden their knowledge, skills and understanding."[2]

The four sub-headings which follow this introduction in the draft Programmes of Study are not recognisable as the key underpinning mathematical concepts which pupils need to understand. "Competence in mathematical procedures, creativity, appreciation of mathematics and critical understanding in using mathematics"[3] are important but they are not key concepts in mathematics. These four themes are repeated, to some extent, in some of the other Programmes of Study, but this is not universal. For example, the Key Concepts in the Geography Programmes of Study **do** seem to be the key concepts underpinning the study of geography: place, space, scale, interdependence, environmental interaction, physical & human processes, cultural understanding & diversity.

Since it has been possible to have dedicated Key Concepts for some of the subjects, there is no obvious reason why this should not also be done for mathematics. Indeed, it is especially important to identify and emphasise the key concepts in this core subject when there is increasing freedom for schools to adapt the curriculum to local circumstances.

The heading "critical understanding in using mathematics" is ambiguous and the further detail and explanatory notes are rather vague and confusing. The phrase could mean either "the important things that should be understood in order to use mathematics" or "understanding and judging what is appropriate and justifiable when using mathematics". The former seems to be what the Key Concepts were intended to be and the latter is touched on in the Key Process "Interpreting and Evaluating" but the importance of checking whether the method and/or solution makes sense in the context is not brought out; this process is essential to mathematical functionality. For example, the calculation $30 \div 4 = 7.5$ is interpreted quite differently when deciding how many four seater cars are needed to transport 30 people from the way it is interpreted when deciding how many sweets each of 4 people can have from a bag of 30 sweets, shared equally.

Key Processes

When the Programme of Study is printed the first two Key Processes, Representing and Analysing, are on one page then there is a blank left hand page before the other Key Processes. This is confusing because the Key Processes make sense as part of a modelling cycle which is incomplete without an interpretation of the analysis. It is not clear whether "Use appropriate mathematical procedures" is part of the Key Process "Analysing" or another Key Process. An indication that the Key Processes are all essential components of a problem solving cycle would clarify matters and help non-specialist and inexperienced teachers in interpreting the Programmes of Study.

What should be taught in mathematics lessons?

The essential purpose of the Programmes of Study in Mathematics is to specify what teachers are statutorily required to teach. The draft Programmes do not contain sufficient detail to fulfil this purpose. It is not entirely clear whether the explanatory notes are statutory or not; if the former, it would usually be clearer to include them in the main body of the text; if the latter, it is somewhat surprising to find that percentages are optional at KS3.

Schools will not want to leave students unprepared for National Curriculum tests, both for the students' wellbeing and because teachers and schools will still be judged on the outcomes of KS3 tests. The level descriptors are largely the same as for the 1999 National Curriculum; these level descriptors will become the effective Programme of Study. The KS3 Programme of Study needs to contain enough detail to enable schemes of work based on it to be sufficient preparation for KS3 tests.

GCSE examination specifications will determine what is taught at KS4. The KS4 Programme of Study as it stands is insufficiently detailed to ensure that all Mathematics GCSEs are of the same standard of difficulty and cover sufficiently similar content to allow smooth progression to A-level. This should be dealt with by having suitable national criteria for GCSE Mathematics. It may simplify matters to make the content of the KS4 Programme of Study the same as the compulsory content of GCSE Mathematics; if the latter is more prescriptive than the Programme of Study then the

lack of detail in the Programme of Study gives an unreal impression of freedom in what should be taught.

Specific Comments on detail

- KS3 & 4 note on "A tool for solving problems and a discipline with distinct structure": the note only refers to mathematics as a tool for solving problems; either the heading or the note should be amended. Although the note refers to professional mathematicians, the examples given relate to mathematicians using mathematics as a tool rather than to doing mathematics for its own sake.
- KS3 & 4 note on "History of mathematics" includes the phrase "an appreciation that pure mathematical findings sometimes precede practical applications"; it is difficult to think of examples of this from the KS3 & 4 content.
- KS3 & 4 note on "Mathematics of different cultures" does not include the importance of mathematics as a universal language that transcends culture; this is in the importance statement but should also be referred to here.
- KS3 & 4 note on "Current applications of mathematics" – the examples given are difficult for teachers to keep abreast of; some national resources to help with this would be useful.
- KS3 & 4 note on "A situation or problem can be represented using mathematics" should refer to exploring a situation and discovering its underlying structure.
- KS3 & 4 note on "Take account of wider factors" should refer to statistics as this is the area of mathematics where students are most likely to encounter these ideas. For example, although the probability of winning the jackpot on the national lottery is very close to zero, people still take part because the potential win is so very much greater than the potential loss.
- It is unfortunate that the only reference to ICT in the main Programme of Study is "use accurate notation, including correct syntax when using ICT" with all other references being in the explanatory notes. This fails to recognise appropriately that the availability of ICT has changed the emphasis for learners of mathematics from their ability to perform algorithms accurately to their understanding of the underlying concepts. In view of the recent investment in ICT in schools, it is neither necessary nor desirable to write Programmes of Study which can be taught without any use of ICT at all (if the notes are non-statutory).
- KS3 & 4 note on "Questioning, analysing and evaluating" states "It is important to be aware that mathematics can be used to inform and misinform." This is a rather bald statement that would be more easily understood with some exemplification. Moreover, the important process of questioning whether a mathematical solution to a problem is a sensible one is not mentioned and it should be.
- The wording under the Key Process of Analysing is "appreciate that there are a number of different techniques that can be used to analyse a problem" at KS3 and "identify a range of techniques that could be used to tackle a problem" at KS4 – perhaps "tackle" is also more appropriate at KS3.
- KS3 note for "Record methods" states "This includes representing the results of analyses in several ways (for example tables, diagrams and symbolic representation)." It is not always necessary to use more than one representation

and students should not be encouraged to engage in unnecessarily repetitive work. The word "various" rather than "several" is better.

- The introduction to Range and Content includes the sentence "The study of mathematics should enable students to apply their knowledge, skills and understanding to relevant real-world situations." This neglects the importance of using real-world situations to help students' understanding of mathematics.
- The Range and Content for Number and Algebra contains "rules of arithmetic applied to calculations and manipulations with rational numbers" for KS3 and "rules of arithmetic applied to calculations and manipulations with real numbers" for KS4. Area and circumference of a circle are included at KS3 as is Pythagoras's theorem so it needs to be "real numbers" for both KS3 and KS4.
- The Range and Content for Number and Algebra at KS3 needs to include reference to prime factorisation as this is important for students' fluency with number as well as being a precursor to algebraic factorisation.
- The explanatory note for "Algebraic expressions" in Range and Content for Number and Algebra at KS3 is badly worded and incomprehensible.
- The Range and Content for Geometry and Measures for KS4 includes the bullet point "conversions between measures and compound measures"; this is ambiguous. Two separate bullet points "conversions between measures"; "compound measures" would be better.
- KS3 & 4 note in Curriculum Opportunities referring to "Other subjects" mentions "enterprise and economic well-being"; this is not a subject, it is part of PSHEE.

References

1. *Evaluating mathematics provision for 14–19-year-olds*, Ofsted, May 2006
2. *KS3 Mathematics Programme of Study for consultation*, QCA, 2007
3. *KS3 Mathematics Programme of Study for consultation*, QCA, 2007