A Level Mathematics & Further Mathematics

An investigation into the reasons for increased uptake in 2009

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Introduction

The numbers of candidates taking Mathematics and Further Mathematics A Levels have risen steadily since 2003, the year in which records were first kept separately for the two qualifications. Figures from 1989 onwards are provided in Appendix A.

This year's increases were particularly large, both in absolute numbers and as percentages. In this respect, mathematics differed from the other STEM subjects, as shown by the figures in Table 1. Clearly something special is happening with mathematics.

Biology	- 0.9%	Computing	- 7.1%
Chemistry	+ 1.9%	ICT	- 2.7%
Physics	+ 4.8%	Technology	+ 0.3%
Mathematics	+ 12.2%	Further Mathematics	+ 15.2%

Table 1 Changes in A Level uptake in STEM subjects 2008-2009

It is very important to understand the reasons for these large increases, including why they are specific to mathematics, so that appropriate measures can be taken both to sustain them and to promote further increases in the years ahead. To this end, ACME decided to collect information from various sources. MEI offered to conduct an online survey of teachers' views and some of the information in this report also features in ACME's findings.

Before moving on to the main body of the report, some cautionary remarks are in order.

The focus of the report is those students who completed their A Levels in the summer of 2009. This cohort entered sixth form in September 2007; this was a time of prosperity, a full year before the start of the present recession and so explanations that relate to the current economic downturn must be irrelevant.

Determining people's motivation is intrinsically difficult; sometimes you, yourself, do not fully understand why you took a particular course of action. Furthermore, the situation in question is complicated with many contributory reasons for what is going on, most of them interacting with each other.

So, it is possible that, despite our best efforts, the best we can achieve may be a partial understanding of the situation.

The survey

The survey was widely publicised by mathematical societies and organisations and the awarding bodies. As a result some 180 responses were received, covering 636 teachers in institutions that taught 7738 students in 2009. This represents over 10% of A Level Mathematics students. MEI would like to say a sincere thank you to all those who advertised the survey or who took part in it.

There were two free response boxes and these produced many interesting comments from respondents. Some of these are quoted in italics at appropriate places in this report. As well as asking about possible explanations, the survey also covered possible double counting and expected uptake in 2010.

Double counting

There has been concern that the numbers reported in 2009 were inflated by changes in certification procedures, with some students featuring in both the 2008 and 2009 figures.

Double counting of A Level Mathematics candidates would typically occur when a student completes and certificates A Level Mathematics at the end of Year 12, takes Further Mathematics in Year 13 and then asks for both to be certificated at the end of Year 13, with more combinations of units now available to Mathematics. However, most schools and colleges do not use this pattern of delivery. Instead they teach the AS Levels in both Mathematics and Further Mathematics in Year 12 and then do the A2 units to complete both the A Levels in Year 13; this leaves no scope for double counting.

Consequently, it seems unlikely that many A Level Mathematics students were counted twice and this was confirmed by the survey. Some 73 students from 20 institutions were identified as being in this category. This would suggest that double counting may explain about 1% in the reported increase of 12.2%; certainly it would seem safe to claim an increase of over 10%.

> *We do have students who have gained modules for A-level Maths in Yr 12, but they do not certificate until the end of Yr 13. Only certificating for A-level Maths once.*

The situation with A Level Further Mathematics is clear cut. Virtually all students complete the qualification in Year 13 and so they have only one opportunity for certification, leaving no possibility of double counting. The reported increase in uptake of 15.2% can be taken entirely at face value.

However, at AS Level the situation is much more complicated with many possibilities for double counting. No attempt was made in this survey to investigate the reliability of the very large AS Level increases reported in August 2009 but it is reasonable to assume that they can partially be explained by double counting. However, the data in Table 2, on the next page, include estimated numbers completing their A Levels in 2010, showing considerable increases from 2009. These students have not come from nowhere; there must also have been genuine increases in student numbers at AS Level in 2009.

Trends

The survey showed the upward trend in both qualifications to be continuing as shown in Table 2. The percentage increases reported for 2008-2009 are compared to those that actually occurred. This shows that, despite being a self selecting sample, the schools and colleges that responded had on average almost exactly the national increases over that period.

Data Source	Survey		UK 2008-9	Survey		UK 2008-9
Year	**Maths**	**Change**	**Change**	**FM**	**Change**	**Change**
2008	6895	-	-	1189	-	-
2009	7738	+ 12.2%	+ 12.2%	1364	+ 14.7%	+ 15.2%
2010 Est	8512	+ 10%	-	1613	+ 18.3%	-

Table 2 Number of students certificating for A Level Mathematics and A Level Further Mathematics
(Notice these figures refer to the full A Levels and not AS Levels)

The numbers reported for 2010 represent real students who are in their schools and colleges now, and so are very encouraging.

> *The numbers this year at AS are very large - 346 for AS and 48 for Further Maths AS. Far more have stayed on into U6th - 155 plus Further Maths students. Resits are at 18. This is over 530 students - over 40% of the college population study maths in some form or other. We did take on more new staff on a part time basis – but even these have had their time increased to cope with the numbers. I hope that the universities can cope with the possible increase in numbers continuing with the subject. Some are already asking for AAA and including A2 in Further Maths – who in year 11 know that this is the subject for them and so select A2 Further Maths? I know that great students are being rejected by some universities for the wrong reasons.*
>
> *Although numbers in year 13 have dipped this year, we have an incredible 21 students in current year 12 with 5 opting for AS Further Maths. How many of these will last is a big question, but it's good to have the numbers to start with.*
>
> *Mathematics and Further Mathematics are the first and second most-popular A-level at this school. Year Group sizes range from 170-180 students, so over 80% take A2 Maths.*

Explanations

The questionnaire for the survey listed 15 possible explanations for the increase in uptake and asked respondents to tick those that they considered most important; it was suggested that respondents choose 6 of them.

The list of 15 possibilities was compiled by asking a variety people, including ACME members and Outer Circle and MEI staff, for their explanations. They were presented in a random order.

The questionnaire asked how many people's views were represented in any response, and the preferences have been weighted accordingly. In some schools and colleges it had clearly been discussed at departmental meetings.

> *We, as a department, would vote for any government which valued the subjects studied for entry on the university courses which lead to graduates for the jobs that the country actually needs as opposed to pretending that all were of equal difficulty.*

However, the explanations actually covered 4 different aspects of the situation and so they have been grouped together accordingly.

A: Establishing a favourable environment

B: The mathematics curriculum

C: In-house reputation

D: Career progression

Although there were marked differences in the support given to the different groups, they are actually all essential parts of a single process. Consequently, it does not seem appropriate to set them against each other; that would be rather like asking which is the most important part of a car, wheels, engine, body or controls; all of them are necessary.

By contrast, the ranking of explanations within groups is based on comparison of like with like. In the commentaries on the various groups that follow, bullet points are used to identify the relevant explanations. They are given in rank order, with the most popular first. In several cases the ranking seems particularly important and so is highlighted.

Readers of this report may wish to consider different approaches to interpreting the data, and so the weighted scores for the various explanations are given in Appendix B.

Group A Establishing a favourable environment

Three of the explanations come in this group and they be described as underlying drivers. They support and, in some cases, underpin many of the explanations in the other groups.

- Enhancement and enrichment events and resources are influencing more students than before.
- The Further Mathematics Network has been a catalyst for greater student engagement with mathematics.
- The media are presenting mathematics more positively.

These explanations are all about creating an environment in which mathematics can flourish; they develop students' interest in it and their awareness of the value of taking it post-16.

> *Creating a positive environment to learn maths at our school.*

The explanations in this group are particularly hard to compare with those in the other groups because their effect is longer term and at a deeper level; it is likely to be cumulative over a period of years. So it is perhaps unsurprising that it was the lowest scoring group.

These explanations are about a change of culture. In many other parts of the world mathematics is seen as one of the good things of life and, whether a choice is available or not, it is regarded as obvious that students should continue it for as long as they are in school.

> *We have had a huge increase in the number of foreign (mainly Chinese) students all of whom study maths/further maths.*

The recent increases in uptake show that we have taken some first steps along this road, but we have a long way to go. The sort of activities that are covered in this group of explanations will be essential if the advances we have made are to become embedded and to lead on to further progress.

> *The students enjoy Maths and want to carry it on! (even if they don't need it).*
>
> *Maths teachers who are seen as cool rather than nerdy.*
>
> *We have an increased number of ex-Engineers teaching Maths, inspiring students to switch to STEM subjects at A-Level and beyond.*
>
> *Maths is particularly seen as a necessary adjunct to Economics; and the perceived difficulty of the subject gives it street value.*

Group B The mathematics curriculum

This group of explanations is about the mathematical experience students have or can expect in their school or college. They are essentially about the mathematics curriculum. Overall this group was ranked above the middle.

Four explanations come within this group, two of them related to the A Levels and the other two about the transition from GCSE. The two pairs are looked at separately here, starting with those about the A Levels.

- The present syllabus for Mathematics and Further Mathematics is fit for purpose and is working well.
- In our school or college, there has been an increase in Further Mathematics and this has improved the image of mathematics generally.

There was particularly strong support for the first of these explanations, about the present syllabus, which was ranked 4th overall and top in this group. Many comments expressed support for it and opposition to proposed changes.

The current structure of A Level Maths and Further Maths is working very well! I fervently hope that QCA will see sense!

We strongly support the current assessment pattern of 6 units for Maths and Further Maths whereby some units can be used within both specs; this makes Further Maths easier to offer. Also, 4 unit models for A-level Maths are likely to cause problems for the organisation / availability of applied units.

Major specification changes are not the way to go.

If the proposals go ahead, the number of students taking & succeeding on AS/A2 maths/further maths courses will decline.

Flexibility in the choice and timing of modules make the subjects more widely attractive.

We have boosted the numbers doing FM AS and A level by teaching 9 modules in the curriculum time of 6 to enable students to gain an AS in FM. This has increased the number prepared to do the additional 3 modules in year 13 to gain the full A level in FM (12 students in 2010. This will be the highest number since 1999).

We are unable to offer Further Mathematics ourselves in-house and depend on external provision for one student.

Further Maths was introduced at AS level 2 yrs ago (one student did the full A level through independent study). The full A level has been introduced for Y12 in Sept 2009.

We set the A level groups and the top set does AS Further Maths at least with the option of the full Further Maths if they want.

The two explanations about the transition from GCSE also caused considerable comment.

- Students are coming through from GCSE with a more positive experience of mathematics.
- The transition from GCSE to the start of AS Level (ie C1) is smoother than it used to be.

There was strong support for the first of these two explanations, about students now having a more positive experience of mathematics at GCSE. This was 5th in the overall rankings. However, there was rather less support for the idea that the transition from GCSE is now smoother; although this explanation received quite a number of votes, it also evoked mostly adverse comments, including some relating it to negative effects of two-tier GCSE.

> *Increased enjoyment of the subject at GCSE.*
>
> *Maths is often one of their best GCSE results, so they think they will be able to do A level.*
>
> *I am utterly shocked to see the suggestion that a possible reason for the increase is a smoother transition from GCSE to AS! I have given students the same diagnostic test for 4 years and the cohort gets worse and worse, this year almost half of our new AS students scored less than 40% in a test of basic GCSE algebra. The schools are under pressure to produce grade Cs not produce students who are ready to take on AS.*
>
> *The two tier GCSE system has allowed more students to gain the entry requirement of a B. Many of these students would have previously been on the Intermediate tier and not have been able to do the A-level at our school. The consequence of this is a larger and weaker cohort at AS. Unfortunately many of these students find the demands of AS beyond them - the two tier system has a lot to answer for!*
>
> *The numbers of students with grade B's at GCSE has increased dramatically since the two -tier GCSE was introduced. However, in many of the feeder schools, due to the pressure of league tables etc, much of the A* material has never actually been taught to the students, so they are less likely to be familiar with the quadratic formula, factorisation, completing the square etc. This has the effect of a high percentage of students with a grade B at GCSE actually dropping the course in the early stages due to difficulty with the material (within the first 4 weeks).*

Group C In-house reputation

These explanations related to the reputations of mathematics departments and to the actions that the staff were taking to encourage mathematics.

- The profile of mathematics in our school or college has improved.
- Students are being encouraged by older siblings (and their friends) who have achieved success in mathematics at this level.
- The standard of post-16 teaching has improved in our school/college.
- We are now making more effort to recruit students into mathematics.
- There has been a widening of participation (eg more students with grades B and C at GCSE).

Perhaps as a result of teachers' modesty, these explanations were not highly ranked.

> *The quality of Maths teaching at our school is very good and pupils are realising that the "so called" easy options will not have the same "clout"... if you have an A Level in Maths people say "you must be clever", when was that last said about Media Studies for example?*
>
> *We get high numbers doing A level because of consistently successful GCSE results and the confidence students have in the provision we make at A level.*
>
> *Much improved results at GCSE. Large increase in A*/A grades resulting in students being much more positive and confident about A level Maths.*
>
> *We do extra support lessons for the less able students at both AS & A2 levels. This has led to better grades at the lower end. The students value this and the students lower down the school (11-19 school) know that this occurs from their older siblings.*
>
> *Change of Head of department and Headmaster to a Mathematician.*
>
> *Since arriving at the school as Head of Dept, I have sold maths and further maths to all students I teach, starting as early as Y7. The results have been improving and we now achieve very good A2 results. This is communicated to students, possibly by other students but they are also far more aware of which subjects offer the best opportunities later.*
>
> *We are a maths and computing college, we have fantastic results and these encourage students to take on maths. We also have experimented with putting GCSE B and C grade student onto AS Use of maths, giving them more success, some are then transferring to AS maths in Year 13. Typically, these students would not previously have had as much success at AS maths.*
>
> *We have moved from strict linear setting to a broader approach which has widened the pool of potential students.*
>
> *What is being done to address the poor press on Use of Maths? I think it is a critical course to widen participation further in the subject, with students who would otherwise not be able to continue with any maths.*

Group D Career progression

The highest ranked explanations were the three that were related to students' university and career prospects.

- More students are seeing mathematics as a subject that will give them a competitive advantage when seeking a university place or employment.
- Students are more conscious of the likely benefits of the subjects they choose in terms of future earnings and career prospects.
- Some universities are looking for more mathematics from prospective entrants.

The first two of them received far more votes than any other explanation.

The third, relating to university entrance requirements, was also highly ranked at 3rd. Given that it only applies to some universities, an important question is whether there would be further increases in uptake if more universities were explicit in asking their applicants for more mathematics.

These explanations are all about the end of the process rather than the mechanisms by which 16-year old students coming out of GCSE and choosing their sixth form subjects are now better informed about their career and university prospects. So they certainly do not tell the full story. Nor is it obvious that they explain the differences between mathematics and other STEM subjects.

> *The top Engineering, Maths and Economics courses at universities now increasingly require Further Maths as a subject but will not accept Maths and Further Maths as two separate subjects. All our Further Maths students complete it as a fourth A-Level.*
>
> *Competition for university places. Some offers say maths is "desirable" but competition makes it almost compulsory if you want to be sure of an offer.*
>
> *Media description of some courses as soft options; recession focusing minds on more traditional courses.*
>
> *Mathematics is seen increasingly as a useful subject for supporting future HE courses eg medicine, psychology, the sciences.*

Comparison with other STEM subjects

This survey has provided many interesting insights that can help to explain the present increases in mathematics uptake. However, they do not give a full explanation of the difference between mathematics and the other STEM subjects. So what has happened differently in mathematics from other subjects?

There would seem to be two answers: the history of the current syllabus and the Further Mathematics Network. It is the view of the authors that, while the many explanations discussed in this report are very important in their own right, it is principally these two influences that explain the difference between mathematics and the other STEM subjects.

The history of the current syllabus

The mathematics syllabus for Curriculum 2000 proved highly unsuccessful and so a new syllabus was introduced in 2004. This did not happen in any other subject.

The 2004 syllabus was specifically designed to overcome the problems of low uptake and retention produced by its predecessor. The priorities which dictated its design were not the same as those that applied to the syllabuses for other STEM subjects, either for Curriculum 2000 or for the recent revisions.

Many respondents commented on the success of this syllabus and the figures support them. The first A Level cohort completed in 2006 and in the 3 years since then uptake has increased by 30%.

The Further Mathematics Network

The government has given Further Mathematics special support in the form of the Further Mathematics Network (now the Further Mathematics Support Programme). This has been highly successful; since its roll-out in 2005, the uptake of A Level Further Mathematics has doubled. This is a much larger increase than has occurred in Mathematics, which in turn has seen much larger increases than other STEM subjects. There has been no equivalent programme in any of the other STEM subjects.

The team running this programme has always seen promoting mathematics in general and particularly its study beyond GCSE as an important part of their role. They are convinced that the benefits of their work are not just with Further Mathematics but cascade down through the age groups, fostering much more positive attitudes towards mathematics.

“More than any other A level subjects, Mathematics and Further Mathematics underpin successful study at degree level of a range of other subjects, and ultimately increase young people’s earning power. The strong growth in numbers taking both these subjects is a tribute to the work of the Further Mathematics Network and others who are working to increase the popularity of Mathematics – and a tribute to the good sense of the young people and their parents who realise that choosing to continue the study of Mathematics is a career-enhancing decision”.

Professor John Holman, Director National Science Learning Centre

Appendix A A Level Mathematics Numbers 1989 – 2009 (Source JCQ)

Year	Mathematics entries (FM excl)	FM entries	Total Mathematics entries (FM incl)	FM as % of Mathematics	Total A Level entries	Mathematics (FM incl) as % of total entries
1989			84 744		661 591	12.8
1990			79 747		684 117	11.7
1991			74 972		699 041	10.7
1992			72 384		731 024	9.9
1993			66 340		734 081	9.0
1994			64 919		732 974	8.9
1995			62 188		725 992	8.6
1996			67 442		739 163	9.1
1997			68 880		777 710	8.9
1998			70 554		794 262	8.9
1999			69 945		783 692	8.9
2000			67 036		771 809	8.7
2001			66 247		748 866	8.8
2002			53 940		701 380	7.7
2003	50 602	5315	55 917	10.5	750 537	7.5
2004	52 788	5720	58 508	10.8	766 247	7.6
2005	52 897	5933	58 830	11.2	783 878	7.5
2006	55 982	7270	63 252	13.0	805 698	7.9
2007	60 093	7872	67 965	13.1	805 657	8.4
2008	64 593	9091	73 684	14.1	827 737	8.9
2009	72 475	10 473	82 948	14.5	846 977	9.8

Table 3 Uptake of Mathematics and Further Mathematics 1989-2009

Note Mathematics and Further Mathematics have only been reported separately since 2003.

Appendix B Popularity of the different explanations

Responses to question 1 are summarised in the table below, with the explanations ranked according to popularity.

Rank	Explanations	Responses
1.	More students are seeing mathematics as a subject that will give them a competitive advantage when seeking a university place or employment.	492
2.	Students are more conscious of the likely benefits of the subjects they choose in terms of future earnings and career prospects.	476
3.	Some universities are looking for more mathematics from prospective entrants.	307
4.	The present syllabus for Mathematics and Further Mathematics is fit for purpose and is working well.	302
5.	Students are coming through from GCSE with a more positive experience of mathematics.	276
6.	The profile of mathematics in our school or college has improved	266
7.	In our school or college, there has been an increase in Further Mathematics and this has improved the image of mathematics generally.	222
8.	Students are being encouraged by older siblings (and their friends) who have achieved success in mathematics at this level.	173
9.	The transition from GCSE to the start of AS Level (ie C1) is smoother than it used to be.	148
10.	The standard of post-16 teaching has improved in our school/college.	140
11.	Enhancement and enrichment events and resources are influencing more students than before.	116
12.=	The Further Mathematics Network has been a catalyst for greater student engagement with mathematics.	109
12.=	We are now making more effort to recruit students into mathematics	109
14.	There has been a widening of participation (eg more students with grades B and C at GCSE).	96
15.	The media are presenting mathematics more positively.	95

Table 4 Ranking of possible explanations for increase in uptake