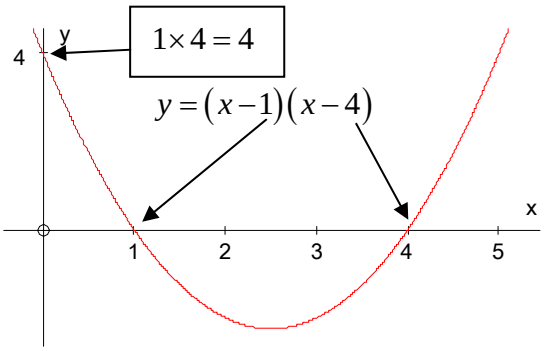
Suggested ideas for using ICT in AS Core

Algebra

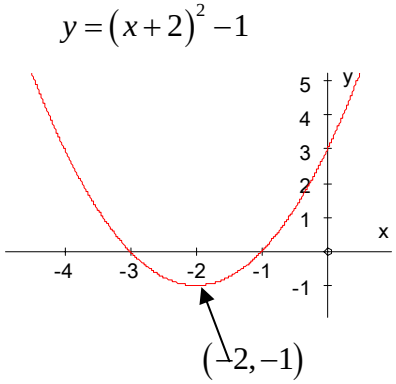
Quadratics

- Graphic calculators or Autograph can be used to think about the link between:

the factorised form of a quadratic and the axes intercepts of the graph:



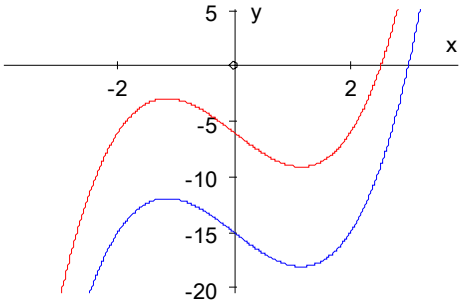
the completed square form of the quadratic and the turning point:



- Similarly, a graphical approach to simultaneous equations will prepare the students for Chapter 2 and give students a visual insight into why, with linear and quadratic simultaneous equations, it is necessary to substitute back into the *linear* equation.

Graphs

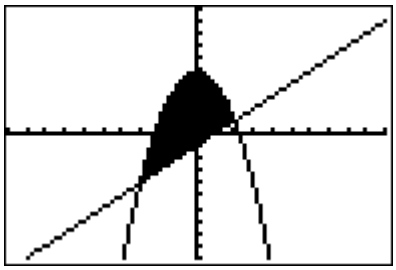
- There are also opportunities to investigate a range of other questions supported by a graph-plotting device:
  - Can a quartic equation have three roots?
  - Given a straight line $y=px+q$ and a quadratic $y=ax^2+bx+c$ what can you tell about the axes intercepts of $y=(px+q)(ax^2+bx+c)$?
  - Plot the graph of $y=x^2$. Investigate the each of the following in turn, using several values of a and explain the effect it has :
    $y=x^2+a,\qquad y=(x+a)^2,\qquad y=a\times x^2,\qquad y=(ax)^2$
- In Autograph, the f(g) button allows you to define a function, say $\mathrm{f}(x)=x^2$. You can then enter equations $y=\mathrm{f}(x)+k$, $y=\mathrm{f}(kx)$ etc. The slow plot mode allows you to compare these graphs.
  - If the curve $y=\mathrm{f}(x)$ touches but does not cross the $x-$axis at the point $x=a$ what do you know about the factors of $\mathrm{f}(x)$?
  - Draw the graphs of $y=(x^2+3x+5)(x-3)$ and $y=\mathrm{f}(x)=x^3-4x-6$. Explain the link between these graphs and the value $\mathrm{f}(3)$



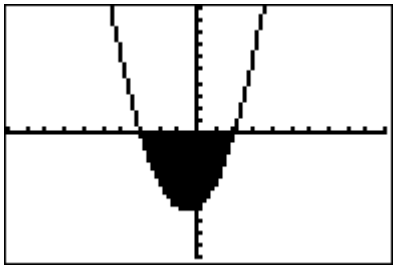
- Choose another non-linear function $\mathrm{f}(x)$ and plot the graph of $y=\mathrm{f}(x)$. As above investigate each of the following in turn:
  $y=\mathrm{f}(x)+a,\quad y=\mathrm{f}(x+a),\quad y=a\,\mathrm{f}(x).\quad y=\mathrm{f}(ax)$

Inequalities

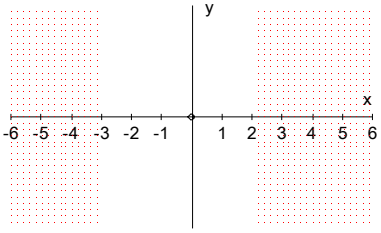
- Graphic calculators can be used to illustrate inequalities such as $x-1<5-x^2$ …



… which can be compared with the simplified version $x^2+x-6<0$ (or $(x+3)(x-2)<0$) …



… whereas Autograph shades the unwanted values of x:



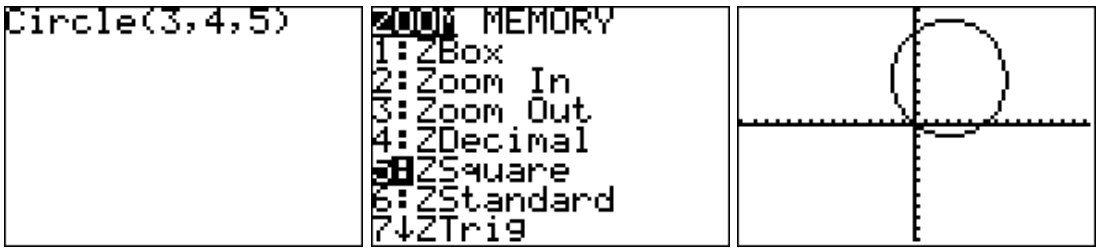
Coordinate Geometry

Equation of a line

- Use a graph-plotter or graphical calculator to plot a line through two points, then work out the equation of the line and check if it is correct.
  On the TI-83Plus, LINE (2,4,5,3) will draw a line through the points (2,4) and (5,3).

Circles

- You can use a graph-plotter or graphical calculator to illustrate the affect a, b and r have in the equation of the circle $(x-a)^2+(y-b)^2=r^2$, or analyse the points of intersection of a line and a circle.
  http://www.meidistance.co.uk/pdf/circlesspreadsheet.xls provides a dynamic approach to teaching circles.
- Also on the TI-83Plus experiment with circle from the draw menu:

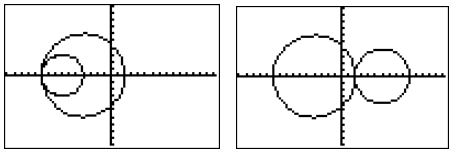


What features of Circle(3,4,5) meant that the circle passed through the origin?

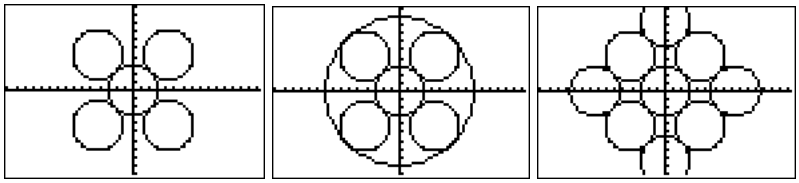
Describe what Circle(-3,-4,5) would look like then check using your calculator.

Find another circle which passes through (a) the origin, (b) the point (3,2).

Draw two circles which touch but not at the origin:



Draw other touching circle patterns:



The constant controller in Autograph also gives the student a dynamic understanding of the role played by coefficients and constants in the equations of curves. See http://www.mei.org.uk/files/pdf/Const_Control_Autograph.pdf

Sequences and Series

Binomial

- Students could be encouraged to write an excel spreadsheet to generate Pascal's triangle (cell B2 = Cell A2 + Cell B1):

1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	
1	3	6	10	15	21	28		
1	4	10	20	35	56			
1	5	15	35	70				
1	6	21	56					
1	7	28						
1	8							
1								

- Link with Statistics: http://www.meidistance.co.uk/pdf/Binomial.xls could be used to discuss the general shape of the binomial distribution. This also gives a very visual way of displaying the answers to Statistics textbook questions.

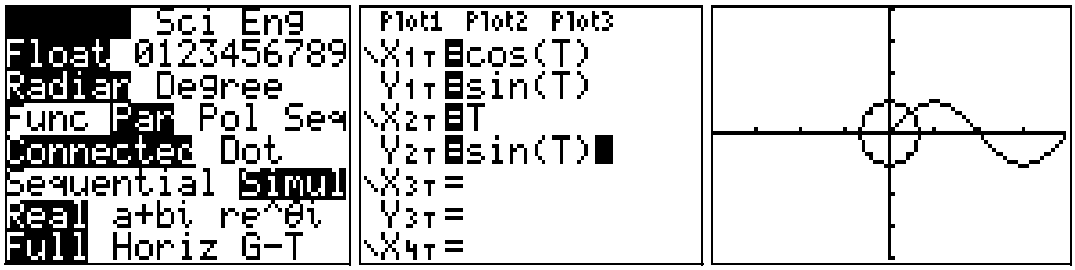
APs and GPs

- http://www.meidistance.co.uk/pdf/ass2ap.xls illustrates the way the first term and common difference affect the graphs of the terms of the A.P. and the sum to n terms against n.
- http://www.meidistance.co.uk/pdf/ass3gp.xls repeats this for G.P.s. clearly illustrating the importance of $|r| < 1$

Trigonometry

Graphs of trigonometric functions

- In Autograph 3 under File>New extras page>Trigonometry the trigonometric functions are plotted from the unit circle. You could also plot $y = \sin^2 x$, $y = \cos^2 x$, highlighting and explaining their similarities, and then $y = \sin^2 x + \cos^2 x$.
- Graphic calculators can be used to draw the trigonometric graphs from the unit circle. For example on the TI-83Plus you could briefly discuss parametric equations and then:

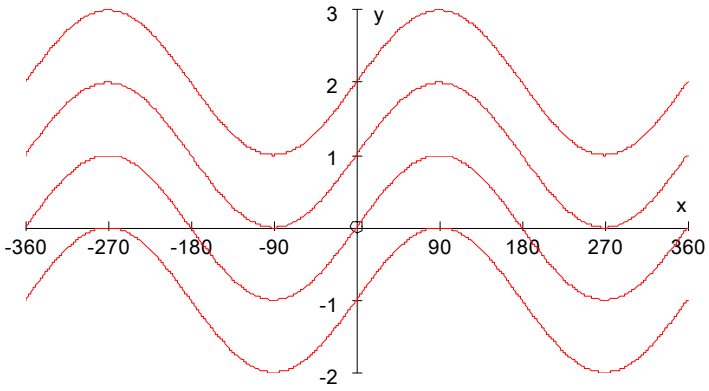


Graphs of trigonometric functions and solving equations

- The spreadsheet http://www.meidistance.co.uk/pdf/Circfns.xls generates the trigonometric graphs and shows how to solve basic trigonometric equations. This is highly recommended!

Transformations of graphs

- Graphics calculators and Autograph are useful when investigating transformations of graphs:

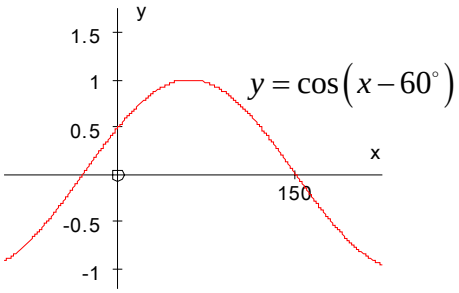


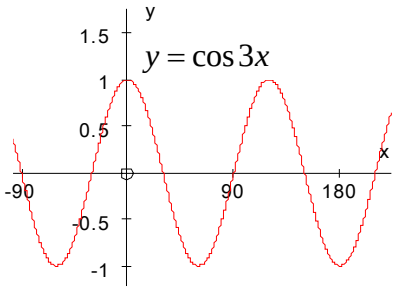
$y = \sin x + 2$

$y = \sin x + 1$

$y = \sin x$

$y = \sin x - 1$



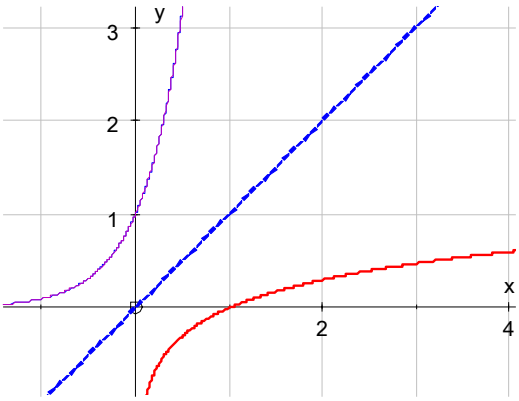


Using the f(g) button define $f(x) = \sin x$ and then enter the equations $y = f(x) + k$, $y = f(kx)$, etc. Using the slow plot mode allows you to compare these graphs.

Logarithms and Exponentials

Logarithms

- In Autograph the reflection in $y = x$ button: allows you to compare the reflection of $y = \log x$ with $y = 10^x$:



- Use the f(g) button define $f(x) = \log x$ and then enter the equations $y = f(x) + \log 2$ and $y = f(2x)$.Using the slow plot mode allows you to compare these two graphs. This can be used to demonstrate the laws of logs.

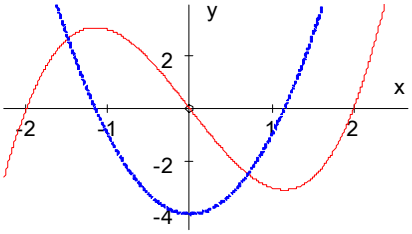
Reduction to linear form

- Excel can be used to generate a table of data and the scatter graph and line of best fit can be added to the spreadsheet.

Differentiation

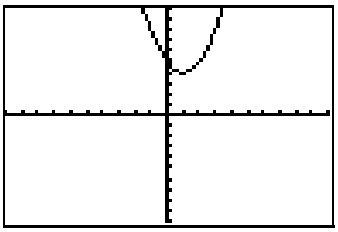
Gradient functions

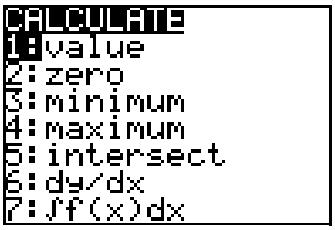
- Autograph allows you to enter a function ($y = x^3 - 4x$ in red) and using the button it will plot the gradient for each value of x (in blue) thereby generating the gradient function curve ($y = 3x^2 - 4$).

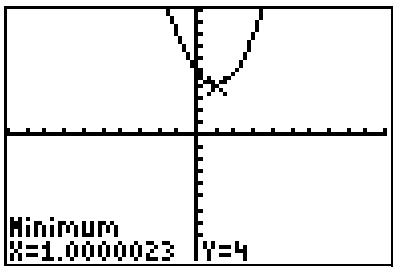


Stationary points

- On Autograph draw a curve, select it and then right-click. **Solve f'(x)=0** will give you the stationary point.
- On the TI-83Plus:
  Draw the graph $y = x^2 - 2x + 5$ From the CALC menu choose 3 to find the minimum value of y:

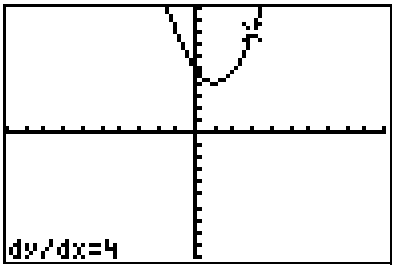


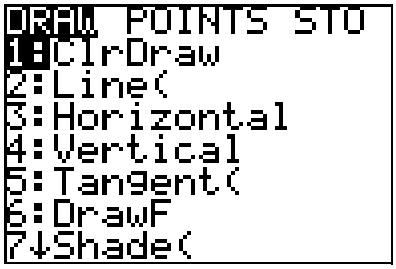


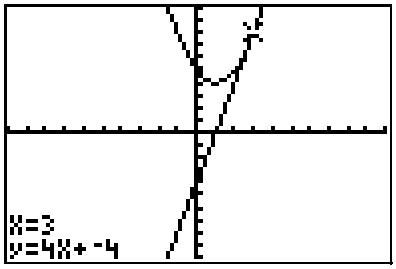


Gradients and tangents

- On Autograph draw a curve, add a point on the curve and then select it. Right-click and select **Tangent**.
- On the TI-83Plus
  Using 6 on the CALC menu … will draw the tangent to the curve at the chosen point giving its equation:
  5 on the DRAW menu… will give the gradient of a curve at a chosen point:



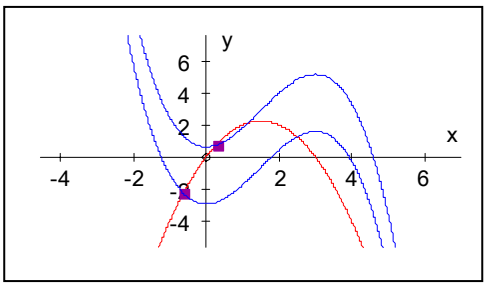




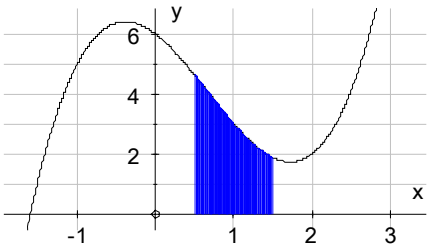
Integration

Finding areas

- In Autograph you can enter a curve ($y = 3x - x^2$ in red) then using the integral function button  click on a point on the curve and it will draw the integral curve passing through this point. In this way it solves the differential equation $\frac{dy}{dx} = 3x - x^2$ passing through a specific point.

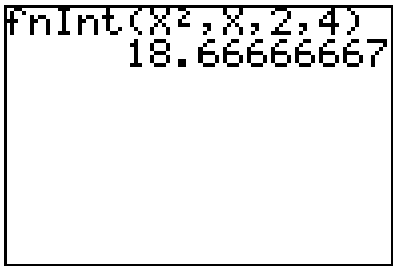


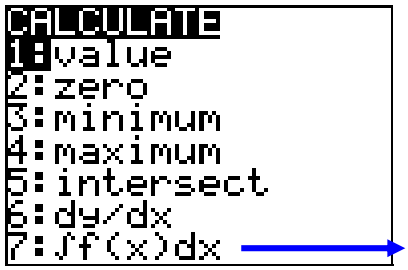
- Also in Autograph you can left click on a curve, then right click and choose the option 'find area'. After selecting the area required it will be displayed in the status bar.

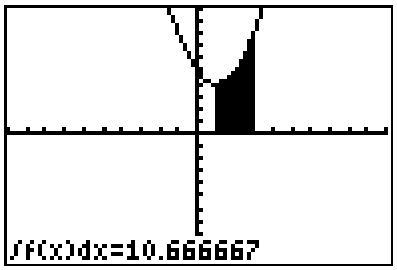


Numerical integration

- On a TI-83Plus







Showing $\int_2^4 x^2\,\mathrm{d}x = \frac{56}{3}$.

Showing $\int_1^3 \left(x^2 - 2x + 5\right)\mathrm{d}x = \frac{32}{3}$.

Autograph

Further examples of the use of Autograph can be seen at:
http://www.autograph-math.com/inaction/