

AEA-style questions in A Level Mathematics papers

A cautionary paper presented by MEI

1. Summary

The February 2005 government white paper, “14-19 Education and Skills”, suggested that A Level examinations should include some harder questions, designed to challenge the most able students.

At A Level we will ... increase stretch for the most able by introducing harder, (AEA)-style questions into separate sections at the end of A Level papers.

This paper looks at three types of issues that would arise if this proposal were to be applied to mathematics.

- Problems associated with the increased overall difficulty.
- Technical difficulties in ensuring that the examinations produce reliable and credible results.
- Doubts over whether the intended benefits would arise.

These issues are covered in Sections 3, 4 and 5 (respectively) of this paper, with the following conclusions.

Fewer students would take mathematics-related courses in Higher Education, and at A Level.

The examination for A Level Mathematics would lose public credibility.

The needs of students would be less well served than they are at present.

Any one of these conclusions would be sufficient to cast doubt on the wisdom of proceeding with the proposal. The three together mean that it would be an act of folly to go ahead with it.

This means that mathematics may well be a special case. That, however, has always been so. From the start of A Levels in the 1950s, the need for special arrangements for the most talented mathematics students was recognised, resulting in it being the only subject with two A Levels, Mathematics and Further Mathematics.

So having different arrangements for the most talented mathematics students would be following a long-standing precedent.

2. Background

2.1 Current provision

In the present curriculum and assessment regime, two different avenues are available to students who wish to do more than the single A Level Mathematics.

- The AEA paper provides harder questions on the A Level syllabus for pure mathematics; no extra content is involved.
- AS and A Level Further Mathematics qualifications are based on extra content. AS Level Further Mathematics is of much the same level of difficulty as AS Mathematics although some of the ideas covered are more sophisticated. By contrast, most A2 Further Mathematics papers include questions that are intrinsically harder than those on single A level papers.

2.2 Fitness for purpose

Implicit in the white paper is the assumption that the present provision is inadequate in all subjects. An alternative view is that in mathematics it is fit for purpose.

- Most AS and A Level Mathematics students find the subject sufficiently demanding as it is, both in terms of difficulty and overall content; they would derive no benefit from harder questions and, as explained later in the paper, would actually be disadvantaged by their inclusion.
- Among those who can benefit from doing more, the majority are better served by learning the extra content of Further Mathematics than by attempting harder questions on work they already know. There is a real danger of these students being given mixed messages.
- The few students who would benefit from harder questions are adequately served by the AEA paper. In its current form the AEA paper, in contrast to the proposal, does not have an adverse affect on other students.

2.3 Student numbers

Several university mathematics departments have recently closed as a result of lack of students. Other subjects that rely on mathematics, for example engineering, are also under pressure. As a consequence, HEFCE is currently launching a project to increase the supply of mathematical sciences graduates.

The basic problem is simply stated: not enough students are continuing mathematics post-16. The number taking A Level is barely half of what it was 25 years ago. The pool of students is just too small to meet the national need for mathematically competent 18-year olds.

Priority should thus be given to measures to increase the numbers taking mathematics post-16. However, as will be explained in this paper, the inclusion of harder questions in A Level papers risks having precisely the opposite effect.

3. Increased difficulty

3.1 Harder papers

The target candidates for AEA papers are the most able students and so questions on these papers are only accessible to the strongest grade A candidates.

The white paper also includes the words

“ ... we will reduce the burden of assessment ”

implying that papers would not be any longer.

Consequently some normal questions would have to be replaced by others that are much harder. This would have several consequences.

- A significant proportion of any paper would be inaccessible to the vast majority of candidates.
- Many teachers would feel obliged to cover the harder questions in their lessons, even though most of their students would be unable to follow the work.
- The papers would be harder overall and so grade thresholds would be lower than at present (unless the new sections are graded separately). This is quantified in Appendix B.

This would result in a design in which grade C would be given with over half the paper wrong and grade E for a very low mark; grade B would probably be awarded at only slightly over 50%.

3.2 Discouraging students from taking mathematics

To understand the effect of hard papers with low thresholds, a little introspection is helpful. Most people believe that there is a ceiling to the mathematics that they can cope with. Thus if a random sample of the population were asked if they could successfully take a mathematics degree, most would say “No, it would be beyond me”. Whether the concept of a personal ceiling is valid or not is less important than the fact that people believe it to be the case.

All students take mathematics up to GCSE. Beyond that they have a choice of whether to continue the subject to A Level, and, if they do so, on into university. A critical element in their decision-making is whether they think they will be able to succeed or whether they will hit their ceiling.

Those who continue with mathematics are taking a gamble; they know that they risk ending up with nothing to show for the next few years of their lives.

Students' decisions as to whether to take the gamble involve personal judgement. A key piece of information in this self-assessment is whether they could do the work at the level they have just taken. Their judgements will not, in general, be based on the grades that examination boards have just given them.

Thus an examination paper that includes significant elements that a student cannot do carries the message "Don't continue with mathematics", and the same is also true of any teaching in preparation for it.

The effect of making a portion of A Level papers harder would be to give exactly this message to many more students. It is thus entirely predictable that it would reduce the uptake of mathematics-related degrees at university.

3.3 A Level uptake

It is not, however, only universities that would feel the effect. It would also result in a reduced uptake at A Level.

A consequence of including harder questions in examinations would be a perception among school students that mathematics is a harder subject. This would not be restricted to those doing the subject and finding themselves faced with questions they cannot do and teaching they cannot follow. It would quickly become general knowledge throughout the school, the word on the street.

All available research shows that for most students mathematics is already harder than nearly all other subjects at AS and A Level. Appendix A gives pass rates for AS Level in recent years; it shows that mathematics has one of the lowest pass rates (even though it tends to be taken by stronger candidates).

Including harder questions would merely exacerbate this situation. An important consideration in this is that in mathematics the difficulty is built into the questions and weaker students just cannot get started; by contrast, in many other subjects any student can answer the questions but some will not do so particularly well. So the effect of including harder questions will be felt more keenly in mathematics than elsewhere.

Fewer students would take mathematics-related courses in Higher Education, and at A Level.

4. Technical difficulties

It is unclear from the White Paper whether the intention is to incorporate the marks from the harder questions into existing grades or to report them separately, as currently happens with AEA results. Either way there are pitfalls.

4.1 A single grade

In the first case, where the marks from the harder questions are treated in the same way as those from the easier questions, lower grade thresholds would be inevitable, as already described. Many candidates would not attempt the harder questions (particularly if they are identified in a separate section) and so give themselves extra time on the easier ones; these candidates would be at an advantage over those who attempt the harder questions but get nowhere with them.

For all but the very best candidates, it would be good examination technique not to attempt the harder questions. But, if students are going to be told not to do these questions, why have them at all? Such a manifestly poor examination would undermine public confidence.

4.2 Separate grades

If, on the other hand, the results of these questions are to be separately reported, there are still potential problems. The key question is whether the harder questions will be done in the same time as the rest, or given extra time at the end of the normal time.

If the time is to be shared, much the same problems arise as those described above. A candidate can effectively gain extra time by not attempting the harder questions and so is at an advantage compared with someone who attempts them.

If the time is not shared, there are in effect two separate examinations. Answers to the first section would need to be collected in before the question paper for the next section was given out. No doubt many candidates would leave at this point. This scenario is very like having an AEA paper split into 3 or more parts that would be taken on different occasions. It would have no advantage over the present AEA but considerable disadvantages because of the short time slots (see Section 5.3).

4.3 Loss of credibility

Since neither option is remotely satisfactory, it has to be concluded that would be impossible to construct the examination so that it would be both reliable and command public confidence.

So, as well as all the concerns raised in Section 3 about the effect on the uptake of mathematics, there is the not inconsiderable drawback that it cannot be sensibly examined anyway.

The examination for A Level Mathematics would lose public credibility.

5. The rationale for harder questions

5.1 A Level Mathematics and Further Mathematics

From time to time in recent years there have been suggestions that the single A Level Mathematics should include harder questions. Usually these have come from university mathematics departments.

These suggestions have been based on two assumptions: that A Level mathematics is designed for those going on to read mathematics at university, and that it is the only mathematics available to such students. The first of these has never been true and the second is now also false.

Only about 5% of those taking A Level Mathematics end up reading mathematics at university. For most of the remaining 95%, mathematics is a service subject, supporting their other A Levels and their future university degree courses; it is for these students that A Level Mathematics is, and always has been, designed.

By contrast, Further Mathematics, particularly at the full A Level, is designed for those going on to read mathematics and closely related subjects.

Until recently it was the case that many students did not have access to Further Mathematics and so were restricted to the single A Level. That is no longer true. Now that the Further Mathematics Network is becoming established, every student in the country who would benefit from Further Mathematics will have access to it, either in-house or through a local Further Mathematics Centre.

So, those university mathematics departments asking for more challenge to be written into A Level Mathematics papers should instead be encouraging, even requiring, their applicants to offer Further Mathematics.

5.2 Grade A in A Level Mathematics

In the last few years rather a high proportion of those taking A Level Mathematics have come out with grade A when the Uniform Marks for their 6 modules have been aggregated.

This has sometimes caused problems for admissions tutors, leading to calls for the later papers to be made harder at grade A. However, such calls are based upon a misunderstanding of what is really happening.

The proportion of candidates obtaining grade A on a typical A2 module is only about 20%. How, then, is it possible for 35% of candidates to obtain grade A overall? The answer is that the aggregation system allows them to bring forward very considerable credit above the grade A threshold from their AS papers. Thus it is possible for someone to score very highly on the AS modules and only moderately at A2 but still come out with an overall grade A at A Level.

Issues relating to aggregation are discussed in more detail in Appendix C

5.3 Mathematical skills

It is also argued that those who have taken A Level Mathematics do not have all the skills they will need to read mathematics at university. Undoubtedly this is true since the single A Level has, rightly, never been designed for such students.

However, it should also be recognised that there is a mismatch between the extra skills required and those that could be tested in the proposed new questions.

At the most these questions might be allocated 30 minutes on each of a number of papers, and probably less. While some candidates may see an appropriate way to tackle such a question straight away, many more will need to try a number of approaches before finding one that will work, and for them 30 minutes may not be adequate. This ability to find your way into an unknown problem ranks highest among the skills that are being asked for, but it requires time.

Thus the proposed new format could hardly be less suitable as a means of fostering the required skills. By contrast, the AEA paper is 3 hours long and so does allow candidates time to make false starts, to learn from them and to develop lines of thinking. So the change from one long paper to questions on several papers would actually replace the existing provision with one that is much less fit for purpose.

However, there has to be doubt as to whether examination questions can in themselves produce the desired results. These are more likely to be achieved by students spending more time on mathematics, meeting some more advanced topics and being well taught. The first two, at least, of these will be achieved by their taking Further Mathematics.

The needs of students would be less well served than they are at present.

6. Conclusion

This paper has highlighted some of the dangers associated with introducing harder questions into A Level Mathematics papers.

A Level Mathematics is just beginning to recover from the debacle of Curriculum 2000 when attempts to toughen demands resulted in a very large fall-off in student numbers and consequent closures of university departments. The very last thing that is needed now is a further attempt to achieve the much the same aim. If it is pursued, similar outcomes will ensue.

The mathematics curriculum already has a unique level of provision for the most able students. What is needed now is for all concerned to do all they can to ensure that students take up the opportunities that are available to them.

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Appendix A AS Rankings, 2001 - 2004

Subject	2001		2002		2003		2004		2005	
	Rank	Fail	Rank	Fail	Rank	Fail	Rank	Fail	Rank	Fail
Welsh	1	2.8	1	2.2	1	0.9	1	1.7	1	1.6
Classics	2	4.0	2	4.2	3=	4.7	4	4.6	4=	4.4
Express Arts	3	4.3	3	4.4	3=	4.7	2	4.0	2	3.5
Music	4	5.2	9=	8.5	8=	8.0	12	8.2	15	8.8
English	5	5.3	5	6.2	5	5.3	6	5.2	4=	4.4
History	6=	6.4	8	7.9	7	7.2	8=	7.0	9	6.1
Media St	6=	6.4	4	5.9	6	5.1	5	4.8	6	4.6
French	11=	9.0	16=	10.4	14	8.8	15	9.1	14	8.6
Spanish	11=	9.0	14=	9.9	11	8.4	11	8.0	10	7.1
Geography	13	9.2	14=	9.9	16	9.2	16	9.7	16	9.7
Business St	20=	13.0	21	13.1	21	13.2	22	13.7	21	13.4
Chemistry	20=	13.0	22	13.3	22	14.4	23	15.1	23	14.9
Sociology	22	13.5	23	14.8	23	14.8	21	13.5	22	13.6
Physics	23	13.9	24	15.4	25	16.0	25	16.5	24	16.1
Biology	26	15.6	26	17.1	27	18.3	27	18.0	27	17.3
Psychology	27	17.2	27	17.3	26	18.0	26	17.9	28	17.9
General St	28	18.1	28	19.9	29	19.3	30	19.3	30	19.0
Computing	29	19.5	30	21.7	31	21.9	31	20.0	31	20.3
Law	30	20.5	29	20.6	28	18.7	28	18.6	29	18.1
Mathematics	31	28.6	31	22.1	30	19.9	29	19.1	26	17.4

Note: The classification of a small number of subjects has changed since 2001. Thus in 2005 the number of subjects reported was 34 and not 31. (The “extra “ subjects are Irish, ICT in addition to Computing, and Further Mathematics.) To the extent that it is possible these figures are based on the 2001 classifications in order to make comparison easier on the same 31 subjects.

Results for AS Further Mathematics were reported in 2004 and 2005 but not previously. This subject has not been included in the rank orders for those two years. The failure rates were 4.6% and 5.3% respectively.

Appendix B Reduced grade thresholds

At present, AS and A Level papers have the following design thresholds.

A: 80%	B: 70%	C: 60%	D 50%	E: 40%
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For a harder question to be true to its description, it would need to be of significant length and so might constitute a quarter of the paper. The effect of replacing one quarter of an existing paper with work which is accessible only to better A grade candidates, would be to lower the design thresholds to about the following levels.

A: 70%	B 52%	C: 45%	D 37%	E: 30%
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Appendix C Aggregation

It is possible that some students who obtain grade A in A Level Mathematics are being treated too generously, but this is by no means certain.

Many of those who take mathematics are very talented. The question is whether the individuals who obtain grade A in A Level mathematics are obtaining higher grades in mathematics than in their other A Level subjects. No data are available on this and research is needed.

If the research were to find that A Level grade A is more generously awarded in mathematics than in other subjects, further research should be carried out into the way the aggregation and grading system performs across different subjects. The system is the same for all subjects but the outcomes would seem to differ from one subject to another. This may well be a consequence of the different patterns of raw mark allocation used in different subjects. For example, in mathematics a mark of 100% is not uncommon, in English it is unheard of.

For many years MEI Structured Mathematics used a different conversion system for converting uniform marks into grades. This was designed specifically to avoid the problem of too much credit above the grade A threshold being taken forward from the easier early papers. The system was effective but it had to be abandoned because it placed MEI candidates in an unfair position in comparison with those taking mathematics with another syllabus.