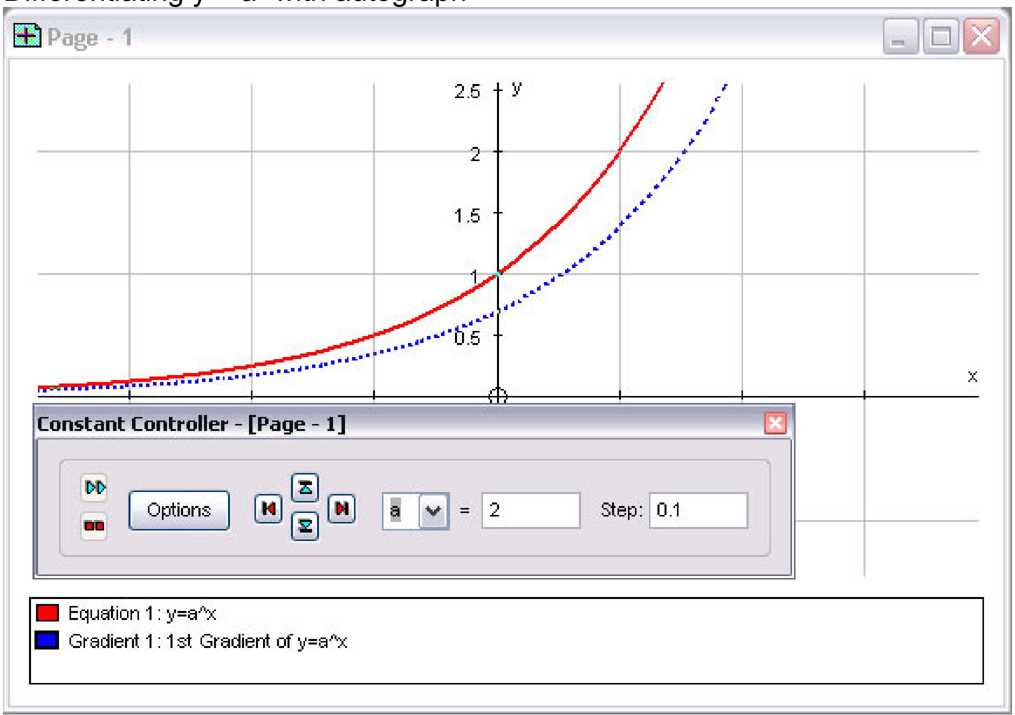
Suggested ideas for using ICT in A2 Core

Core 3

Chapter 2: Natural logarithms and exponentials

The exponential function

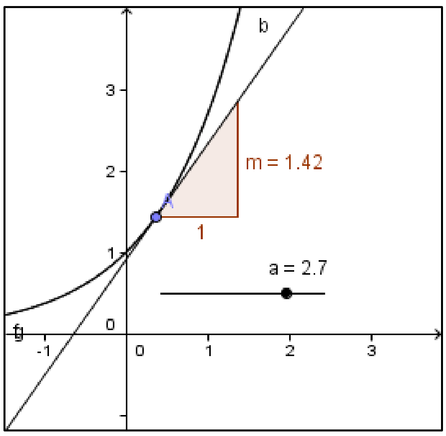
- Differentiating y = a^x with autograph



The value of e can be approximated by plotting $y = a^x$ then adding the gradient function. As the value of *a* is changed with the constant controller students should observe that the gradient function is of the form $\frac{dy}{dx} = k \times a^x$ where the value of *k* depends on the value of a. (For *a* = 2, *k* ≈ 0.7; *a* = 3, *k* ≈ 1.1; *a* = 4, *k* ≈ 1.4: these can be returned to later as *k* = lna). The students should then be able to observe that there exists a value of *a* between 2 and 3 such that *k* =1, i.e. a function whose derivative is itself. Scaling in on the axes and the constant controller leads to a value of approximately 2.72.

- Differentiating y = a^x with Geogebra
  1. Add a slider (which should be "a")
  2. Input y=a^x (you may want to vary a at this point)
  3. Add a New Point on the curve (this should be "A")
  4. Add the tangent at A
  5. Add the slope at A

  As an alternative to points 3-5 you can Input:
  `Derivative[f(x)]`
  or
  `f’(x)`



- Expressing e^x as the sum of terms with Excel.

  Ask students to suggest an infinite polynomial which when differentiated term-by term would remain unchanged. They will hopefully(!) come up with $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$

	A	B	C	D
1	1	n	term	
2		0	1	
3		1	1	
4		2	0.5	
5		3	0.166667	
6		4	0.041667	
7		5	0.008333	
8		6	0.001389	
9		7	0.000198	
10		8	2.48E-05	
11		9	2.76E-06	
12		10	2.76E-07	2.718282

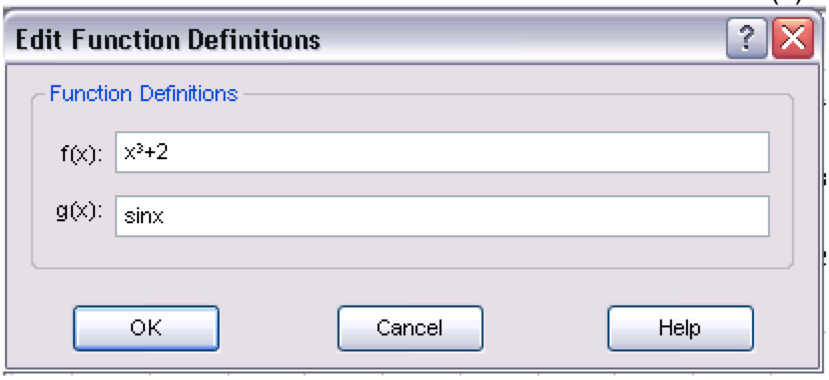
  Enter a value for x in Cell:A1, start with 1.
  Start 2 columns labelled "n" and "term".
  Enter the numbers 0, 1, 2, 3, … in the n column.
  In the first row of the term column enter
  =A1^B2/FACT(B2)
  "Fill-down" the term column with this formula.
  Sum the term column.
  The value of *x* can then be changed and their formula verified.

Chapter 3: Functions

- Investigating transformations of functions with Geogebra
  Functions can be inputted directly into the Input bar. e.g. `f(x)=x^3+2`
  Parameters can be defined by adding sliders.
  Graphs can then be plotted to investigate y = f(x), y = af(x), y = f(bx), y = f(x) + c, y = f(x + d), y = −f(x), y = f(−x) and the values of the parameters can be altered with the sliders.

- Investigating transformations of functions with Autograph

  The function button f(g) allows functions to be entered for f(x) and g(x).

  Graphs can then be plotted to investigate y = f(x), y = af(x), y = f(bx), y = f(x) + c, y = f(x + d), y = −f(x), y = f(−x) and the values of the parameters can be altered with the constant controller.

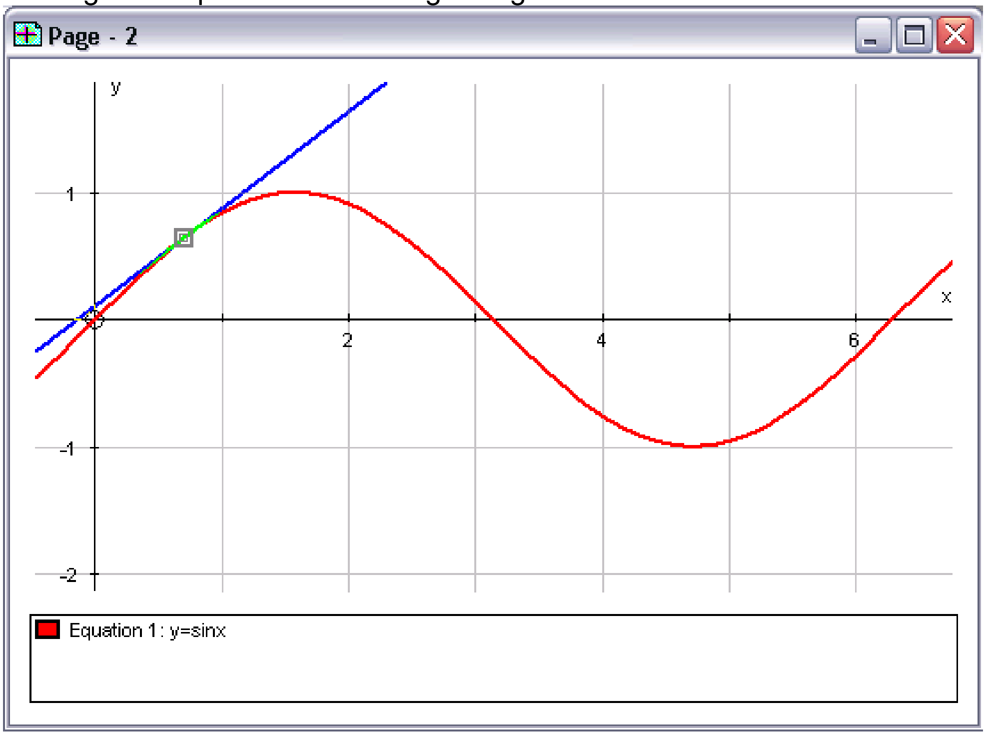
- Investigating inverse functions with Autograph
  Functions can be entered as above and the graphs of y = f(x) and x = f(y) can be compared.

  Alternatively for a graph entered directly (i.e. not as a function), the reflection button, will display the line y = x and the graph reflected in y = x.

  Students can use this to investigate inverse functions.

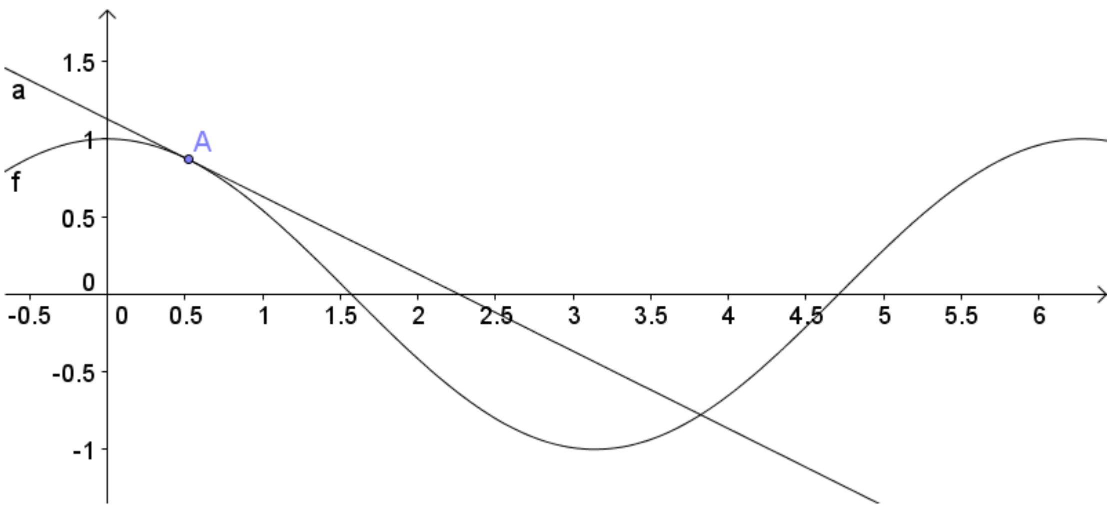
Chapter 4: Techniques for differentiation

- Derivative of sin and cos by observing the tangent on Autograph
  The tangent to y = sin*x* can be displayed by adding a point to the graph and then right clicking on the point and selecting "Tangent".



  Students should then observe that gradient varies from 1 (at 0) to 0 (at π/2) etc. which suggests cos*x*. Students can then investigate cos*x*, *a*sin*x* and sin(*bx*).

- Derivative of sin and cos by observing the tangent on Geogebra
  The gradient of the tangent to y=cos(*x*) can be displayed by adding a curve to the line, drawing the tangent to the curve at the point and then adding the slope at that point.



  NB y=cos(x) will plot the graph with the angle measured in radians and y=cos(*x*°) will plot the graph with the angle measured in degrees.

Chapter 6: Numerical solution of equations

- Excel and Autograph can be used extensively for this – see the online resources.

Core 4 (MEI):

Chapter 7: Algebra

- Use of Excel for evaluating $(1 + x)^n$ for small x.

  Set students the task of produce a spreadsheet that will calculate the first few terms in the expansion of $(1 + x)^n$ for small x and identifying how many terms are needed for the sum to be accurate to a certain number of d.p.

  To do this:
  Label 6 columns: x, n, r, coeff, term and sum
  Enter 0.1 in Cell A2.
  Enter 0.5 in Cell B2.
  Enter the values 0, 1, 2, … in Cells C2, C3, C4…
  Enter 1 in Cell D2.
  Enter =B2 in Cell D3.
  Enter =D3*(B2-(C4-1))/C4 in Cell D4 and "Fill-down" the rest of this column with this formula.
  Enter =D2*A2^C2 in Cell E2 and "Fill-down" the rest of this column with this formula.
  Enter =E2 in Cell F2.
  Enter =F2+E3 in Cell F3 and "Fill-down" the rest of this column with this formula.

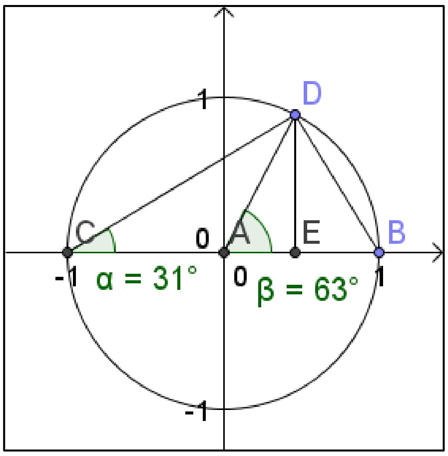
  The values in the cumulative sum column can be compared with the "true" value of $(1 + x)^n$ and both *x* and *n* can be varied.

	A	B	C	D	E	F
1	x	n	r	coeff	term	sum
2	0.1	0.5	0	1	1	1
3			1	0.5	0.05	1.05
4			2	-0.125	-0.00125	1.04875
5	"True" value		3	0.0625	6.25E-05	1.0488125
6			4	-0.03906	-3.9E-06	1.048808594
7	1.048808848		5	0.027344	2.73E-07	1.048808867
8			6	-0.02051	-2.1E-08	1.048808847
9			7	0.016113	1.61E-09	1.048808848
10						

Chapter 8: Trigonometry
Identities

- Double angle formulae in Geogebra
  Define points A:(0,0), B(1,0) and C: (-1,0).
  Use these to draw the unit circle.
  Add a point on the circle D.
  Add a point E:(x(D),0)
  Draw segments AD, BD, CD and ED.
  Add angles ACD and BAD

  The resulting diagram can be used to prove the double angle formulae.



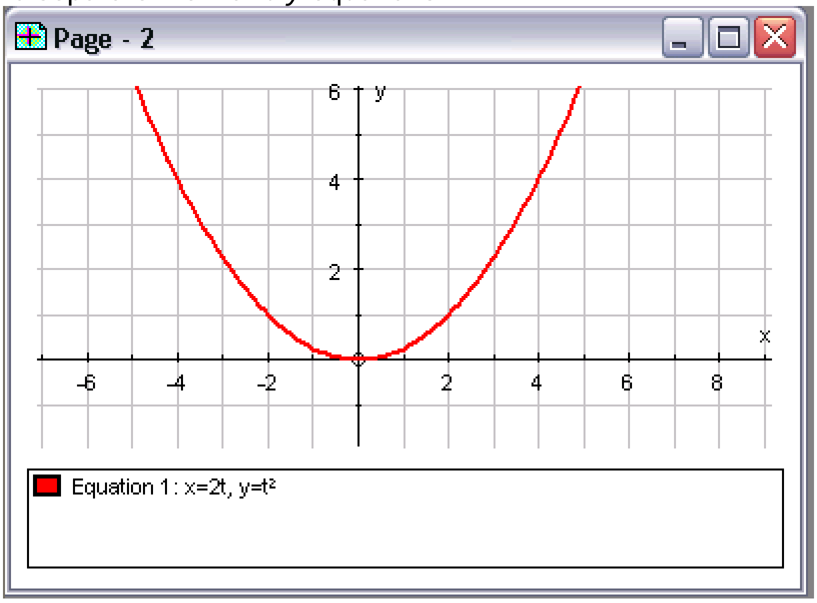
- Graphs – Use of Autograph/Geogebra to discover/display identities
(e.g. can the graph of y = $\cos^2 x$ be transformed into the graph of y = cos2x by means of a stretch and a translation?
Use the constant controller/slider to find values of a and b so that the graph of
y = asinx + bcosx is the same as y = Rcos(x + 30°)

Small angle approximations

- Use of Autograph/Geogebra to find small angle approximations for sin, cos and tan:
$x,\ 1-\frac{x^2}{2}$ and x respectively can be demonstrated/discovered by drawing the tangents to the curves at x = 0 and the range for which they are valid can be demonstrated.

Chapter 9: Parametric Equations

- Parametric equations can be entered into Autograph using the parameter *t* and a comma to separate the *x* and *y* equations.



- Parametric curves can be entered into Geogebra using the syntax:
`Curve[expression for x, expression for y, parameter, lower limit, upper limit]`
e.g. `Curve[2t, t^2, t, -5, 5]`
- Parametric equations can also be entered into most graphical calculators

Chapter 10: Further techniques for integration

- Autograph – Volumes of revolution
See: http://www.autograph-math.com/inaction/td/Volumes.htm

Chapter 11: Vectors

- Vectors on Autograph
See "C4 Vectors with Autograph":
http://www.mei.org.uk/files/pdf/C4_Vectors_Autograph.pdf

Chapter 12: Differential equations

- Differential equations on Autograph
  1st order DEs can be displayed on Autograph by entering
  $\frac{dy}{dx} = ...$
  this then gives a tangent field.

  Clicking on the screen at a point gives a particular solution through that point.

  The tangent fields can be used to explain why
  $\frac{dy}{dx} = ky$ generates an exponential curve and
  $\frac{dy}{dx} = kx$ generates a parabola.

Autograph

Further examples of the use of Autograph can be seen at:

http://www.autograph-math.com/inaction/

Further Information

For further information, including advice about the use of ICT in AS Core see:
www.mei.org.uk/ict/