The Structure of A Level Mathematics

A cautionary paper presented by MEI

Summary

The government White Paper of February 2005 included the following words.

> *Currently, most A levels have 6 units, each separately assessed and sometimes by more than one exam paper. In future, most A levels will have four larger units, covering the same amount of content, but only 4 assessments. This will reduce by a third the assessment burden. It will also reduce costs and address timetabling difficulties. This structure will not be appropriate in every subject and we can expect it to be introduced gradually, as exams are re-accredited.*

This paper sets out the reasons why this structure is not, at least for the time being, appropriate in mathematics. These are summarised in the following bullet points.

- New AS and A Level mathematics specifications were introduced in 2004 for first A Level certification in 2006. Consequently the accreditation of mathematics specifications is current and up-to-date. Re-accreditation is not due.
- Further major changes are anticipated in 2009 following development work on Pathways; it would be extremely disruptive to have a change to a 4-module system closely followed by another major change.
- The primary aim of the 2004 changes was to reverse the decline in the numbers of students taking both Mathematics and Further Mathematics at AS and A Level.
- All the early indications are that the 2004 specifications will successfully achieve this aim.
- Moving to a 4-module structure would be set in motion even before the first cohort of students would have completed A Level on the 2004 specifications, and so without any evaluation of their impact.
- Moving to a 4-module structure would require major changes to the content of AS and A Level specifications.
- Unless the move were accompanied by a substantial reduction in the Subject Core, mathematics would be made significantly less accessible.
- It is entirely predictable that in consequence
  - student numbers would fall
  - it would be harder to retain and recruit mathematics teachers.
- The national interest would therefore be ill served by a change to 4 modules at this juncture.

Background

The first modular A Level, MEI Structured Mathematics, was introduced in 1990 and this had 6 modules for an A Level. The following year two more modular A Levels came on line, London with 4 modules and UCLES with 6. Further 4- and 6-module specifications became available in subsequent years from other awarding bodies.

The two systems ran satisfactorily through the 1990s. There were small advantages and disadvantages to both systems; some people preferred six, some four. Neither had an overwhelming edge over the other.

The situation changed in the year 2000, with the introduction of Curriculum 2000, when only 6-module specifications were allowed.

At that time all subjects were required to have modular assessment. This, together with the expectation that students would take more subjects at AS Level, produced a significant rise in the number of examinations students took and the number of timetable slots required. The move to change from 6- to 4-module A Levels in general is designed to reduced this "assessment burden".

Why mathematics is not an appropriate subject - see the White Paper

The fact that there have been 4-module mathematics A Levels in the past shows that there is no intrinsic reason why mathematics should not have such a structure. However, recent events make this a particularly inappropriate time for such a change, if it is to occur at all.

Uniquely, following the major changes with the introduction of Curriculum 2000, mathematics specifications were changed yet again in 2004. Curriculum 2000 proved such a disaster for mathematics, with a substantial fall-off in student numbers, that it was agreed that urgent action was needed. After considerable thought and discussion, two fundamental changes were introduced and both were designed around the 6-module structure.

- There was a reduction in the overall content while leaving the pure mathematics intact.
- AS Further Mathematics was made into a genuine AS Level that students could do in Year 12.

These changes underpin the whole strategy for increasing the uptake of mathematics at this level. For reasons that are explained in the next two sections, a move to a 4-module structure would effectively mean that they would need to be reversed.

So far the 2004 specifications have run for one year. The first A Level students have yet to complete their second year and so, as yet, there has been no possibility of carrying out a formal evaluation of their impact. (Such an exercise would require data from more than the first cohort of students anyway.) However, informal evidence from teachers suggests that more students are now taking mathematics. This is manifestly not the right time for re-accreditation of mathematics specifications.

The balance between pure and applied mathematics

Until 2004, A Level Mathematics consisted of 3 pure modules, *Pure Mathematics 1*, *2* and *3*, and 3 applied modules (in mechanics, statistics or decision mathematics). The same total pure mathematics content is now spread over 4 modules, *C1*, *2*, *3* and *4*. Only 2 applied modules are now required.

Thus the total A Level content has been reduced by one sixth with the loss of one module of applied mathematics. The reduction has been widely welcomed by teachers on the grounds that covering the work is now more realistic in the available time.

However, the split between pure and applied mathematics has changed from equal amounts of each, to ⅔ of pure and ⅓ of applied. (The same proportions apply at AS Level with 2 pure modules and 1 applied.)

Previous 4-module specifications ran at a time when half of the A Level was pure and half applied mathematics. They had 2 pure and 2 applied modules at A Level, one of each at AS Level. All modules were equally weighted. Clearly the present system cannot be fitted into 4 such modules without changes to the proportions of pure and applied mathematics.

If a move to 4 such modules were not to involve an increase in the total content (and it is widely agreed that any such increase would be highly undesirable), a decrease in the pure mathematics content would be required and an increase in the applied mathematics.

Two difficulties would be associated with such a reduction in the pure mathematics.

- It would require a complete re-write of the Subject Core, which defines the pure mathematics requirement, involving a loss of 25% of its content. The present Subject Core has only recently been agreed for the 2004 specifications; removing such a large amount of it would prove very contentious.
- When the 2004 changes were made, it was argued that maintenance of the pure mathematics content would ensure that standards did not fall; this argument could no longer be sustained.

Further Mathematics

The importance of Further Mathematics, both in covering important content for those going on to read subjects with a high mathematical content at university (for example engineering, science and, of course, mathematics itself), and in providing young people with intellectual stimulation, is now widely recognised. The government, for example, is supporting the Further Mathematics Network.

Until 2004, students were only in a position to start Further Mathematics when they had completed the standard A Level Mathematics; AS Further Mathematics was thus at a totally different standard from all other AS Level qualifications.

This changed in 2004 when AS Further Mathematics was made into a genuine AS Level. It is now accessible to suitable students in Year 12 (i.e. immediately after GCSE) in parallel with the standard AS Mathematics. The key element in this change is a compulsory pure AS Further Mathematics module, *FP1*.

The design requirements for the content of *FP1* are strict.

- It may not duplicate any topic in the Subject Core.
- It may not depend on any content in the A2 Subject Core.
- Additionally, it must be part of a coherent package progressing into the later pure Further Mathematics modules, *FP2* and *FP3*.

Those who designed *FP1* specifications found that there was just enough suitable material for a single module of the present size. To construct a larger module would require padding it out with content that would not be built on in subsequent modules. This would clearly be undesirable.

The new design for AS Further Mathematics has already proved its worth with a 27% increase in uptake in its first year. Those involved in the Further Mathematics Network believe that much larger increases will occur, provided the system is left as it is.

Alternative structures

As well as the previous structure of 2 pure modules and 2 applied, all equally weighted, there are two alternatives which at first sight seem possible but would not in fact prove viable.

Mixed applied and pure modules

One apparent possibility is to include some of the pure mathematics in the applied modules. Thus an AS pure topic would be examined in Mechanics 1, Statistics 1 and Decision Mathematics 1. This would avoid the need to decrease the pure mathematics and increase the applied.

However, the same topic would need to be covered in all three of these AS applied modules to ensure that all candidates covered the core. Consequently, a candidate taking two of them would cover the same topic twice and receive double credit for it. This would clearly be unacceptable from an assessment perspective. The associated repetitiveness of study would also be academically unsatisfactory and a waste of valuable teaching and learning time.

Unequally weighted modules

Another superficially attractive idea is to have double-weighted pure modules. So the present *C1* and *C2* would be put together to form a double module, worth 200 uniform marks; similarly for *C3* and *C4*. All the applied modules would be single weighted at 100 marks.

While this idea could work for the standard A Level Mathematics, it runs into insuperable problems with Further Mathematics.

The difficulty of expanding *FP1* from its present size to ¼ of an A Level has already been discussed. Making it into a double module would be even more mathematically unsatisfactory.

The problem would continue with A2 Further Mathematics where the compulsory *FP2* would need to be made into a double pure module; this would inevitably mean that content which at the moment is not compulsory would have to become so.

Problems for students with unequally weighted modules

It is easy to get carried away by the pursuit of technically viable solutions involving unequally weighted modules and to forget the needs of students. There are several more major disadvantages with them.

Loss of applied mathematics

Even if a double *FP1* module could be put together, it is far from certain that it would be desirable. The 2004 changes reduced the amount of applied mathematics in our curriculum, and so the importance attached to it. However, many Further Mathematics students are able to redress the imbalance by taking more applied than pure modules. This would no longer be possible with double weighted pure modules.

The prospect of this further erosion of the position of applied mathematics gives rise to a number of related concerns, affecting both the standard A Level Mathematics and Further Mathematics.

- There would be a reduction in the scope and richness of the mathematics experienced in sixth forms; more students would take only one applied option.
- In some institutions, teaching mechanics might no longer be considered viable and in consequence their students would be inadequately prepared for futures in engineering and related careers.
- Many able students, particularly those going on to university to read subjects which make substantial use of mathematics as a service subject, would be less well prepared.
- Mathematics would become a less attractive option for those considering taking it as a service subject.

Mathematics would appear less accessible

Double weighted pure modules would be forbidding, particularly for AS students both in Mathematics and in Further Mathematics. Most students would not be ready to take them in January, in contrast to *C1* and *FP1*. Consequently a reduction in student numbers would be extremely likely.

If, as would seem likely, the examination time for the double units were to be less than the total time for the two equivalent present units, the need for syllabus coverage would almost certainly mean a reduction in the emphasis on the basic work in the modules. The average level of the questions would move towards the more advanced work at the end of the modules. If so, this would not only make the papers harder but also mean that essential groundwork was often treated more lightly in the teaching.

The wrong time for new specifications

Earlier sections of this article have explained why new specifications were introduced in mathematics in 2004, and how their whole rationale would be undermined by a change to a 4-module structure. There are other considerations as well.

Pathways

Following Recommendation 4.11 of the Smith Report, studies are now in progress into new mathematics Pathways for 14-19 year-olds. It is anticipated that this work will result in a fundamentally new mathematics curriculum in 2009. Major changes to AS and A Level can be expected then. So any new specifications introduced before then, as a result of a move to a 4-module framework, would be obsolescent before anyone had even studied for them. The Pathways work is unique to mathematics.

Demoralisation of teachers

The 2004 specifications have been well received by teachers. There is a general feeling that they make realistic demands on students and so allow teachers to do an effective job. Consequently, a major change at this stage, before the A2 examinations have even been sat, would be very badly received. Many teachers would see it as a indicative of a lack of respect from those in authority. At a time when there is a desperate shortage of mathematics teachers, a course of action which will make it harder to recruit and retain them would seem foolhardy indeed.

Costs

Schools and colleges have just invested a lot of money in new textbooks for the 2004 specifications. The effect of an early change of specifications would be to write off much of this investment.

There would be other associated costs, particularly in human resources. New examinations and teaching materials involve a great deal of work from many of the most creative people in mathematics education. Such people are left unable to spend their time doing things that would improve and enrich students' learning.

As an example, the very substantial on-line support which has been developed in order to make the Further Mathematics Network possible would all need to be replaced. This is skilled work which would divert people away from ensuring the success of the Network as a whole.

Loss of coherence

Under the present system it was possible to design mathematically coherent modules. When the Subject Criteria were being discussed various possibilities were rejected on the grounds that it would have been impossible to construct good modules, particularly in applied mathematics. As a result, present specifications are, on the whole, well constituted, allowing students to learn the subject reasonably holistically.

A change to 4 modules would inevitably involve moving material between modules, with the danger of compromising the underlying design. It would be foolish to replace what we have by something that is less mathematically sound, but this could happen. Years of work developing successful modules could easily be undone.

AS/A Level student numbers

The number of students taking A Level Mathematics hit an all time low in 2002, following the introduction of Curriculum 2000. It was little more than half the number it had been 25 years previously.

Consequently there is now a dearth of suitable students to go on to university to read subjects with a strong mathematical content, and several well-known university departments have been forced to close or amalgamate. There is also a reduction in the (already low) level of mathematical skills among those entering the workforce. The impact will be felt particularly in the supply of suitable people to work in technological industries or to go into mathematics teaching. The pool of students taking mathematics post-16 is just too small.

The situation, however, looks better following the introduction of the 2004 specifications. There is genuine optimism among teachers that student numbers will increase over the next few years. To introduce a further change before seeing whether the 2004 specifications have indeed lived up to their promise could only be described as irresponsible.

The same is true with Further Mathematics. The new AS Further Mathematics, introduced in 2004, is proving much more accessible and as a result there was a large increase in AS Further Mathematics numbers in 2005. It is entirely predictable that a change to a 4-module structure would have an adverse effect on this improvement.

An adequate supply of young people who are competent in mathematics is much, much more important than conformity to a particular examination structure.

Conclusion

It is easy to imagine that moving from a 6-module to a 4-module assessment structure would be a small, almost trivial, change for mathematics. That is definitely not the case.

We have recently seen, with Curriculum 2000, the damage that can be done to mathematics by introducing changes without adequate thought and preparation. A hurried move to a 4-module structure would carry an extremely high likelihood of a similar national disaster.

Are we prepared to risk university departments continuing to close for lack of suitable applicants for mathematics related degrees, and industry continuing to be unable to recruit employees with the educational background required to compete in today's market place ?

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