Double Award GCSE Mathematics

GCSE 2

Exemplar Paper 3

(Higher Tier)

Time allowed: **2 hours**

Paper total: **120 marks**

Calculator allowed

This paper is one of a set of 6 exemplar papers written by MEI, covering the Foundation and Higher Tiers of GCSE 1 and 2.

The aim of these papers is to inform public discussion. They do not contribute to any existing GCSE qualification.

June 2006

Section A

A1. **You should not use a calculator for this question; you must show your working clearly**

Simplify $2 \times 1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{4} \times 1\frac{1}{5}$

(2 marks)

A2. Change $0 \cdot 2\dot{3}$ into a fraction in its lowest terms.

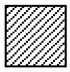
(3 marks)

A3. If you multiply the positive number x by 10 and then add 11, you get the same number as you do when you square x. What is x?

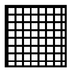
(3 marks)

A4. This is a magic square (the totals of all the rows, columns and diagonals are all the same).

7	**?**	11
?	x	**?**
5	**?**	9

(a) Write in terms of x the value in the cell shaded 

(3 marks)

(b) Write in terms of x the value in the cell shaded 

(1 mark)

(c) What is the value of x?

(3 marks)

A5. Simplify $\dfrac{2x^2+7x+3}{4x^2-1}$

(4 marks)

A6. What is the 100th digit after the decimal point in the decimal expansion of $\frac{1}{7}$?

(5 marks)

A7. A sequence whose nth term is u_n is defined by

$$u_1 = 2 \text{ and } u_{k+1} = \frac{1}{1-u_k}$$

So $u_2 = \frac{1}{1-u_1}$, $u_3 = \frac{1}{1-u_2}$, $u_4 = \frac{1}{1-u_3}$ and so on.

(a) Calculate the values of $u_2, u_3, u_4, u_5, u_6, u_7, u_8$.

(3 marks)

(b) Complete the following statements:

If the remainder is 0 when n is divided by 3 then u_n =

(1 mark)

If the remainder is 1 when n is divided by 3 then u_n =

(1 mark)

If the remainder is 2 when n is divided by 3 then u_n =

(1 mark)

(c) What is u_{1000}?

(1 mark)

A8. Find the coordinates of the points where the line $y = 2x + 3$ meets the circle $x^2 + y^2 = 6$.

(6 marks)

A9. Sue and Tom share a bottle of lemonade. Sue drank 30% more than Tom did. What percentage of the lemonade did Tom drink?

(4 marks)

A10. A cross country course has been measured to be 4700m correct to 3 significant figures. The winning runner completes the course in 22 minutes 13 seconds (to the nearest second).

(a) What is her maximum possible average speed for the course in kmh^{-1}?

(4 marks)

(b) What is her minimum possible average speed for the course in kmh^{-1}?

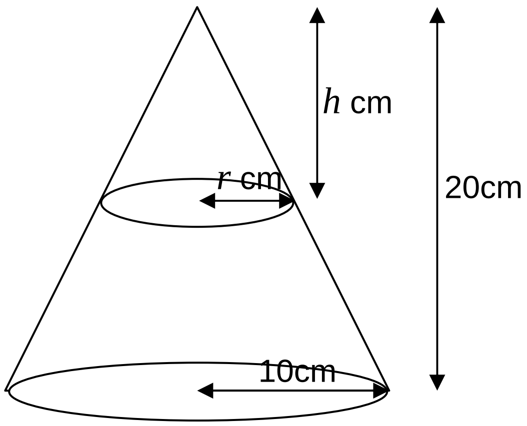
(4 marks)

A11. If p, q, r, and s are 4 consecutive numbers, in order, prove that

$$p + q + r + s = rs - pq$$

(5 marks)

A12.



In the diagram above there are two cones. The base of the large cone has a radius of 10cm and the base of the smaller cone has a radius of r cm. The height of the large cone is 20 cm and the height of the smaller cone is h cm.

(a) Explain why $r = \frac{h}{2}$.

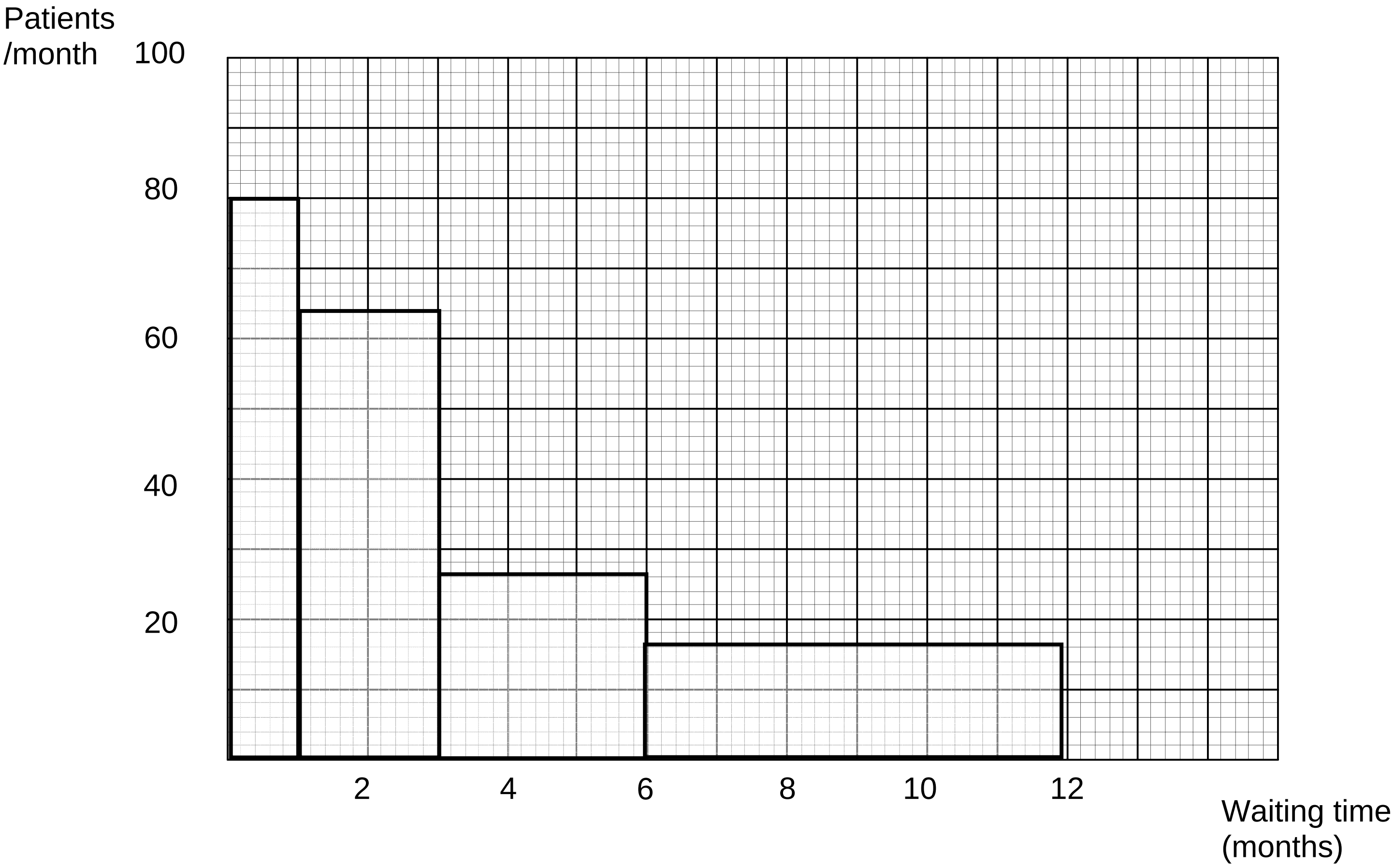
(2 marks)

(b) If the volume of the smaller cone is 50 cm^3, calculate h.

(3 marks)

Section B

B1. The waiting times of patients who had surgery at St. Swithun's Hospital are represented in the histogram below.



(a) How many patients waited between 6 and 12 months?

(2 marks)

(b) Estimate the probability that a patient chosen at random from these data had to wait less than 4 months for surgery at St. Swithun's.

(5 marks)

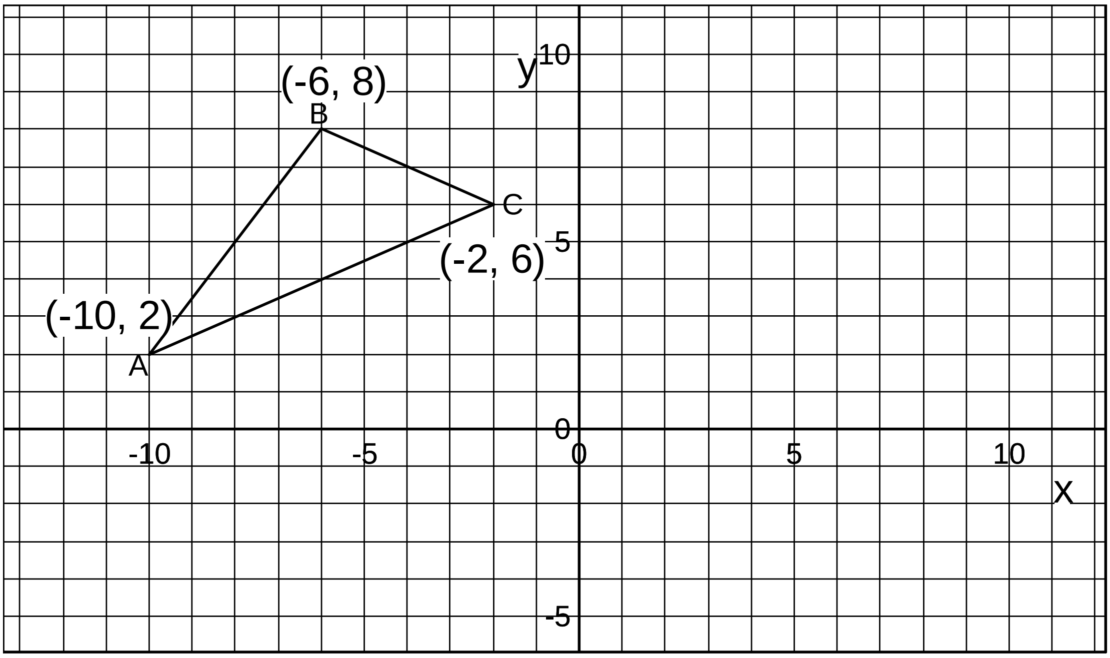
(c) Explain why your calculation in (b) is an estimate.

(2 marks)

(d) Estimate the mean waiting time for these patients.

(6 marks)

B2.



The triangle ABC in the diagram above is moved as follows

- firstly it is rotated clockwise through 90° about A,
- then it is translated through $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$,
- finally it is rotated clockwise through 60° about the origin.

(a) Sketch the locus of B.

(8 marks)

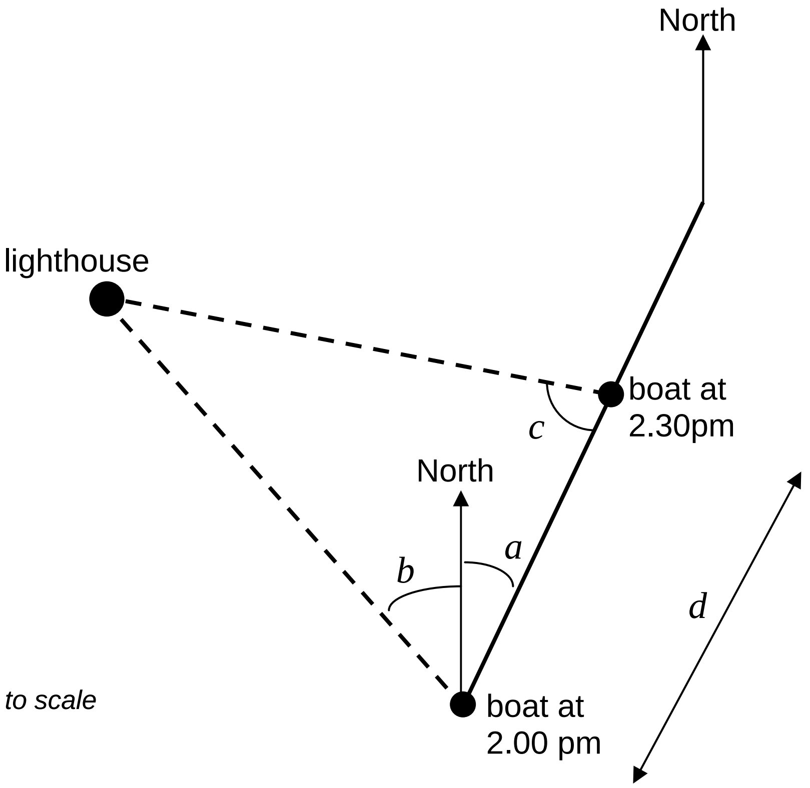
(b) How far does the point B move in total?

(6 marks)

B3. A boat is out at sea in fog. At 2pm, all the crew can see is a lighthouse on a bearing of 320°.

The boat is travelling at a constant speed of 4 kmh^{-1} on a steady bearing of 030°. You may assume that its speed and direction are constant.

30 minutes later the crew can see that the lighthouse is on a bearing of 290°. This is represented in the diagram below.



Not to scale

(a) What are the values of the angles, a, b and c?

(3 marks)

(b) What is the length d?

(1 mark)

(c) How far is the boat from the lighthouse at 2.30pm?

(4 marks)

(d) At what time between 2.00pm and 2.30pm is the boat closest to the lighthouse?

(7 marks)

B4. A hockey team has reached the semi-final of an important cup competition. The team's star striker has nearly recovered from an injured foot.

- The probability that the striker will be fit to play in the semi-final is $\frac{2}{3}$.
- If the striker is not fit in time for the semi-final, she will definitely be fit in time to play in the final.
- If the striker plays in any game, the probability of the team winning is $\frac{3}{4}$ (independent of the opposition).
- If the striker doesn't play, the probability of the team winning is $\frac{1}{2}$ (again, independent of the opposition).
- If the striker plays in the semi-final, there is a probability of $\frac{1}{5}$ that she will get injured and so miss the final.

(a) What is the probability that the team will win the semi-final?

(6 marks)

(b) What is the probability that the team win the cup?

(10 marks)