Ideas for Teaching Moments

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In summary

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Abstract

- This session explores ways to teach moments, including the use of contexts and simple experiments to bring the topic to life!

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Introduction

- Moments is a relatively straightforward topic
- It is an application which many students find somewhat easy, yet one with which a slip can easily be made in more complex questions
- Clear presentation, diagrams and statements of ‘directions’ are key

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Moments in the syllabus

- Moments is in Edexcel M1, all other boards M2 (and beyond)
- *OCR (M2)*

Equilibrium of a Rigid Body

Candidates should be able to:

(a) calculate the moment of a force about a point in two dimensional situations only (understanding of the vector nature of moments is not required);

(b) use the principle that, under the action of coplanar forces, a rigid body is in equilibrium if and only if (i) the vector sum of the forces is zero, and (ii) the sum of the moments of the forces about any point is zero;

(c) solve problems involving the equilibrium of a single rigid body under the action of coplanar forces, including those involving toppling or sliding (problems set will not involve complicated trigonometry).

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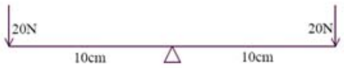
AQA (M2) *(M4 bottom box)*

Moments and Centres of Mass	
Finding the moment of a force about a given point.	Knowledge that when a rigid body is in equilibrium, the resultant force and the resultant moment are both zero.
Determining the forces acting on a rigid body when in equilibrium.	This will include situations where all the forces are parallel, as on a horizontal beam or where the forces act in two dimensions, as on a ladder leaning against a wall.

Moments	Rotations Moment of a Force $= \mathbf{r} \times \mathbf{F}$
Couples.	Understanding of the concept of a couple.
Reduction of systems of coplanar forces.	Reduction to a single force, a single couple or to a couple and a force acting at a point. The line of action of a resultant force may be required.
Conditions for sliding and toppling.	Determining how equilibrium will be broken in situations, such as a force applied to a solid on a horizontal surface or on an inclined plane with an increasing slope. Derivation of inequalities that must be satisfied for equilibrium.

Introduction to the ideas

Consider a light rod as shown.
Is this system in equilibrium?
What force is exerted by the fulcrum on the rod?

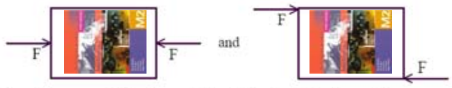


Now move the fulcrum as shown.
Is this system in equilibrium?
The forces are the same.

Clearly not: why not? The turning effect of the right-hand force is greater. What force would we need to apply to the left-hand end to balance it, and what would now be the force exerted by the fulcrum on the rod? Discuss.

Introduction to the ideas

Our Mechanics 1 particle model is now inadequate, because we have to consider the effect of forces which are acting through different points. For example, for this tray, the particle model makes no distinction between



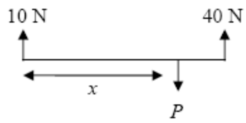
We need to use a **rigid body model**, in which a body has size and shape (which recognizes the effect of turning forces) but does not deform.

Introduction to the ideas (cont)

- The moment of the force, **F**, about an axis through A, perpendicular to the plane containing A and **F**, is *Fd*.
- Many problems can be solved by using a combination of resolving forces and taking moments.
- ***Always draw a diagram –*** if you try to work without a diagram, you are very likely to make mistakes with signs, or to miss out forces.
- ***Remember to include reaction forces at a support or hinge in the force diagram.*** These have no effect when you take moments about the support or hinge, but you need to take them into account when you resolve forces or take moments about a different point.

Basic example

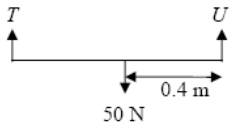
- What is the distance, *x*, required for a light horizontal rod, of length 1.5 m to be resting in equilibrium?



- Resolving vertically:
  $10 + 40 – P = 0$,
  $P = 50$ N
- Taking moments about left-hand end:
  $40 \times 1.5 – Px = 0$
  $50x = 60$
  $x = 6/5 = 1.2$ m

Basic example

- What are the forces *T*, *U* required for a light horizontal rod, of length 1 m to be resting in equilibrium?
- Taking moments about right-hand end:
  $50 \times 0.4 – T \times 1 = 0$
  $T = 20$ N



- Resolving vertically:
  $T + U – 50 = 0$,
  $U = 50 – 20$
  $= 30$ N

Alternatively…(or in addition)

Use lots of contexts!

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Contexts

See-saw

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Standard

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Contexts

See-saw

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Standard with one person either side

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Contexts

See-saw

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Standard with one person either side and one in the 'middle'

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Contexts

See-saw

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Standard with one heavy person and one light

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Contexts

See-saw

2 v 1

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Contexts

See-saw Bookcase!

Moments used in marketing!

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Video action

Doing the calculations 'instinctively'

Contexts

Cranes

This was actually outside the conference building!

Contexts

Cranes

Contexts

(A much more complex example)

What is happening in this video clip?

Investigation

Is Mr Bump safe under this construction?

Where is the mathematics? Can you recreate this, with rulers, books, bricks?

Starter exercise

Here is a man painting a ledge

Discussion point:

Where is the mathematics?

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Starter exercise

Comments may be made on how much paint is needed etc.

Should draw out the health and safety aspects of how the ladder is staying in place!

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Concept

Couples

A couple is represented by two equal parallel forces not in the same line that are in opposite directions.

This system has a zero resultant force and a non-zero moment about any point.

Any good examples?

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Contexts

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Example

- Edexcel M1 Jan 2006 Qu 3

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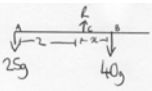
3 a)



EQUILIBRIUM.

Moments about point C: $25g(2) = 40g(x)$

$\Rightarrow x = \frac{50g}{40g} = \frac{5}{4} = 1.25m.$

b) If the beam has uniform weight then its mass/weight can be taken to act through the midpoint C

(which was important for the previous question)

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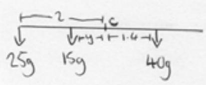
c)



Still in EQUILIBRIUM

Moments about point C:

$40g(1.4) - 15g(y) - 25g(2) = 0$

$\Rightarrow 56g - 50g = 15g(y)$

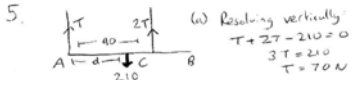
$\frac{6g}{15g} = y = \frac{6}{15} = \frac{2}{5} m$

(or 0.4m)

Example

- Edexcel M1 June 2006 Qu 5

Edx June 06 solution

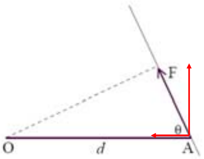


<u>Forces at an angle</u>

Consider a rod OA, of length d, and a force acting at A at an angle θ with OA.

Moment of F about O is $F \times$ the perpendicular distance from O to the line of action of F
$= F\,d \sin\theta$

$= d \times F \sin\theta$
$= d \times$ component of F perpendicular to OA
(the other component of F passes through O and has no moment about O)



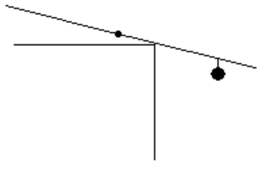
Interesting point of detail

What is the correct force diagram for a ladder (sitting on rough ground) resting against:

-a smooth wall
-a window ledge
-a peg

Investigation

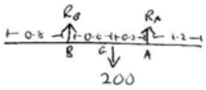
Measuring the mass of a metre ruler



More difficult example

- Example exam question that has ‘forces at an angle’

3 (i)



Taking moments about B ↺ +ve

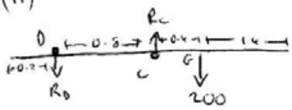
$$200 \times 0.6 - R_A \times 0.8 = 0$$

$$R_A = 150 \text{ N}$$

Resolving ↑ +ve $R_B + R_A - 200 = 0$

$$R_B = 200 - R_A = 50 \text{ N}$$

(ii)

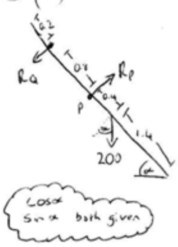


Taking moments about D ↺ +ve

$$200 \times 1.2 - R_C \times 0.8 = 0$$

$$R_C = 300 \text{ N}$$

3 (iii)



Taking moments about P ↺ +ve

$$0.4 \times 200 \cos\alpha - 0.8 \times R_Q = 0$$

$$R_Q = 96 \text{ N}$$

Resolving perpendicular to plank ↗ +ve

$$R_P - R_Q - 200\cos\alpha = 0$$

$$R_P = R_Q + 200\cos\alpha$$

$$= 288 \text{ N}$$

Contexts

Rucksack - how should you pack it and carry it?

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What would you discuss?

Contexts

Exercise

Side plank exercise

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Experiments/investigations

- Mechanics lends itself well to undertaking practical and computer experiments/investigations and contextual examples
- Take the opportunity wherever possible, e.g. those cited earlier in the talk, those cited in earlier sessions

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Key Points

- Equilibrium is a key idea
- Resolving and taking moments are standard practice
- Be more careful when considering ‘forces at an angle’