



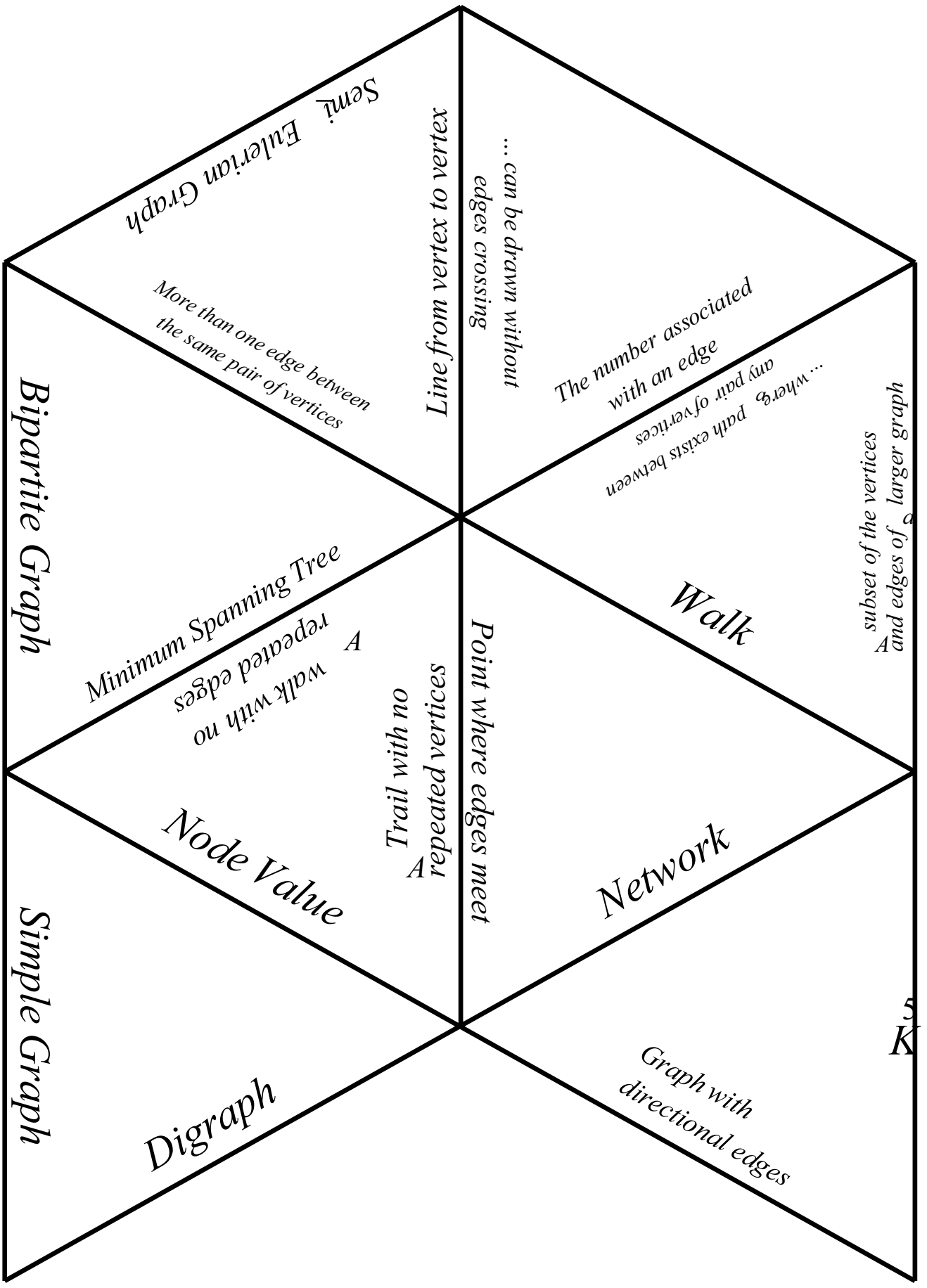
# Solution

Start

A	R	L	A	N	E	B	I	P	A	R
E	C	P	C	Y	C	T	O	R	E	T
D	G	E	L	E	E	N	U	T	T	I
D	E	E	R	T	O	A	W	E	G	R
E	A	I	D	J	A	L	K	D	I	A
G	R	L	A	L	P	M	O	C	H	P
R	T	P	O	E	T	E	I	M	P	L
E	E	L	O	R	E	V	S	A	H	E
N	N	X	E	T	O	D	E	M	N	I
E	C	C	E	U	N	N	A	I	O	A
C	T	E	D	L	E	R	I	L	T	N

Finish

1. A R C
2. E D G E
3. P L A N E
4. C Y C L E
5. T R E E
6. D E G R E E
7. L O O P
8. T R A I L
9. A D J A C E N T
10. B I P A R T I T E
11. R O U T E
12. W A L K
13. D I G R A P H
14. C O M P L E T E
15. V E R T E X
16. C O N N E C T E D
17. E U L E R I A N
18. N O D E
19. S I M P L E
20. H A M I L T O N I A N



Semi-Eulerian Graph

Line from vertex to vertex

... can be drawn without edges crossing

The number associated with an edge

... where path exists between any pair of vertices

subset of the vertices and edges of a larger graph

Bipartite Graph

More than one edge between the same pair of vertices

Minimum Spanning Tree

walk with no repeated edges

Walk

Point where edges meet

Trail with no repeated vertices

Node Value

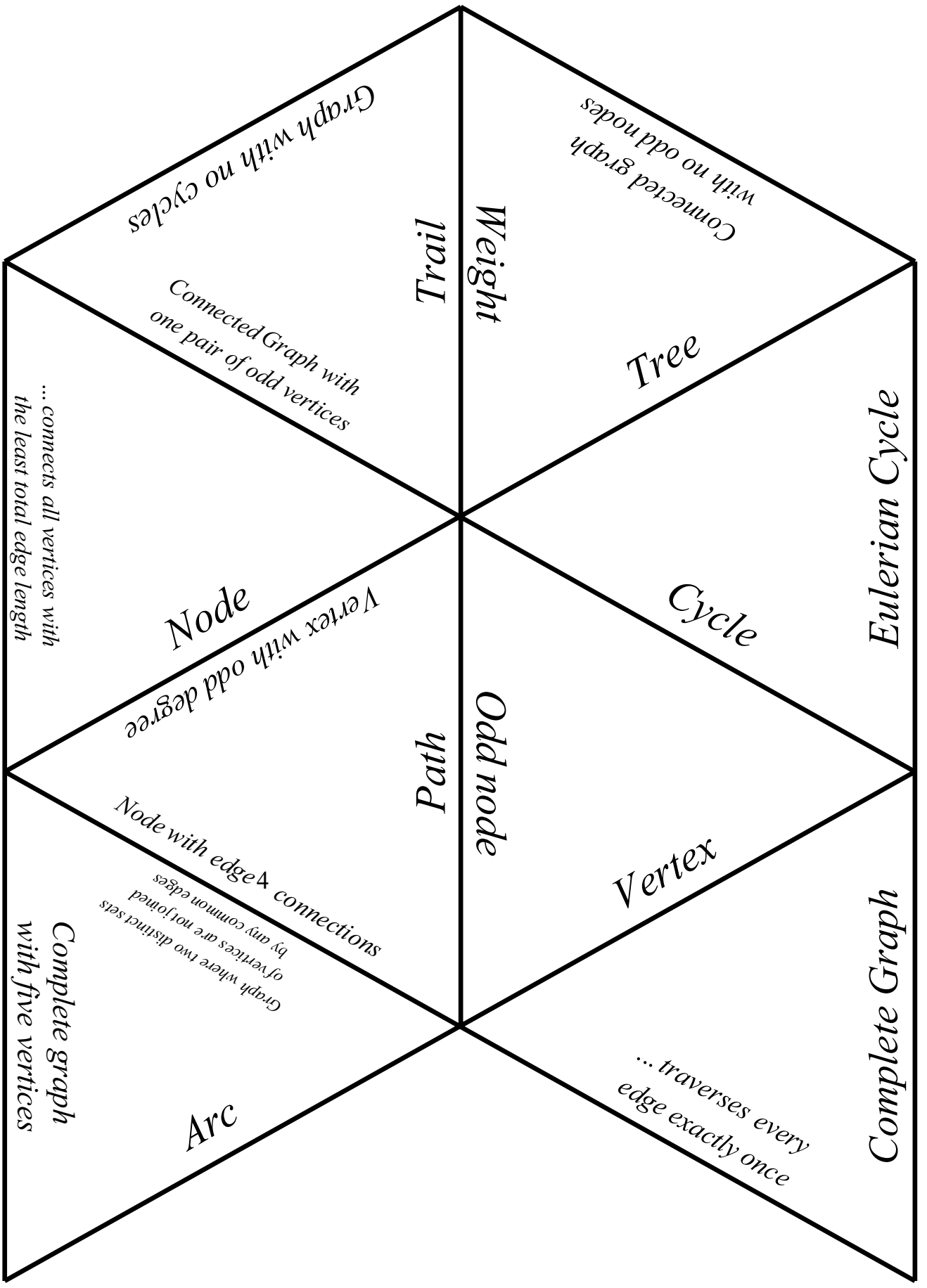
Network

Simple Graph

Digraph

Graph with directional edges

5  
K



*Graph with no cycles*

*Connected graph with no odd nodes*

*Trail*  
*Weight*

*Connected Graph with one pair of odd vertices*

*Tree*

*... connects all vertices with the least total edge length*

*Node*

*Vertex with odd degree*

*Eulerian Cycle*

*Cycle*

*Path*  
*Odd node*

*Node with edge  $k$  connections by any common edges of vertices are not joined*

*Vertex*

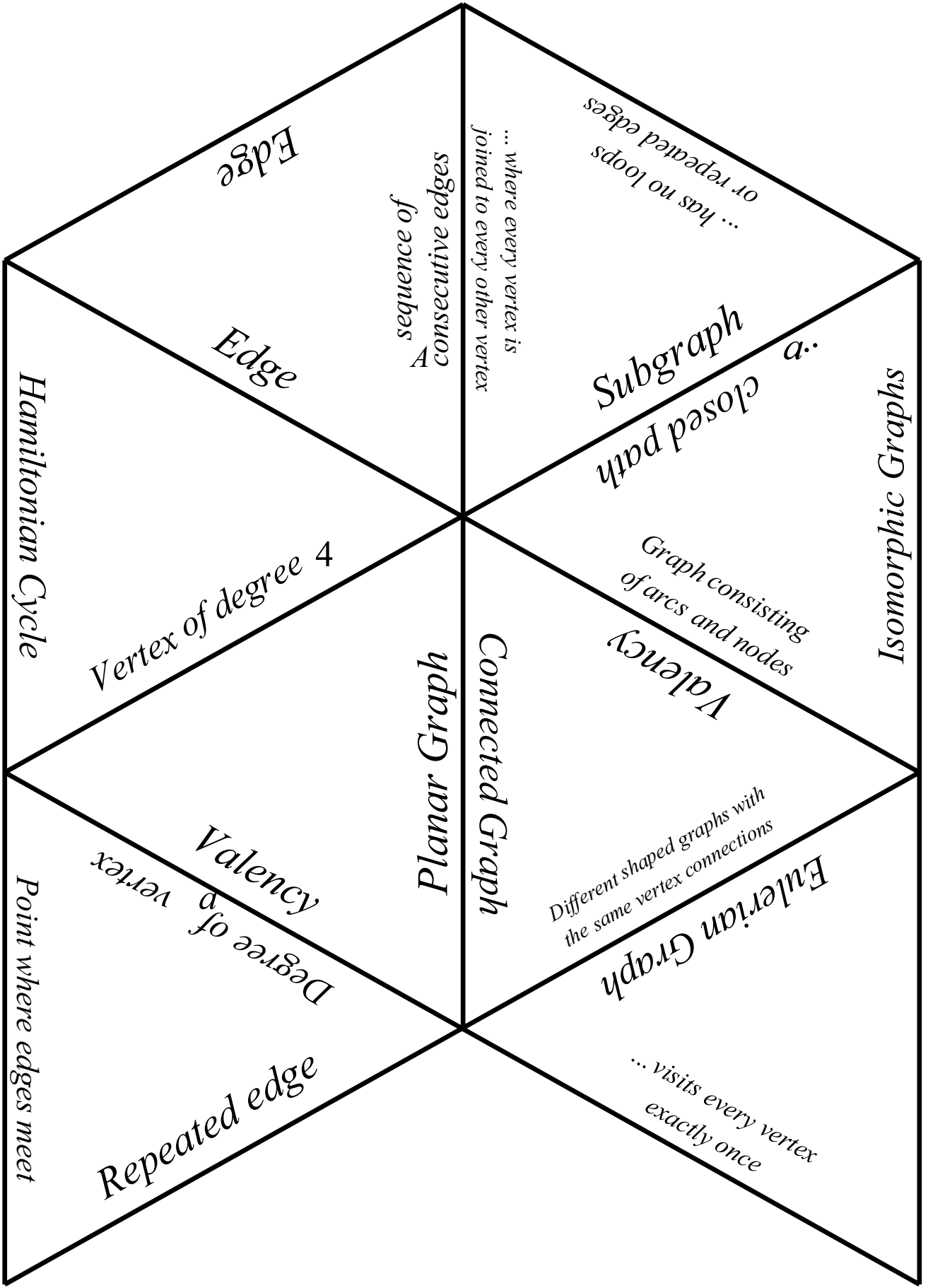
*Complete Graph*

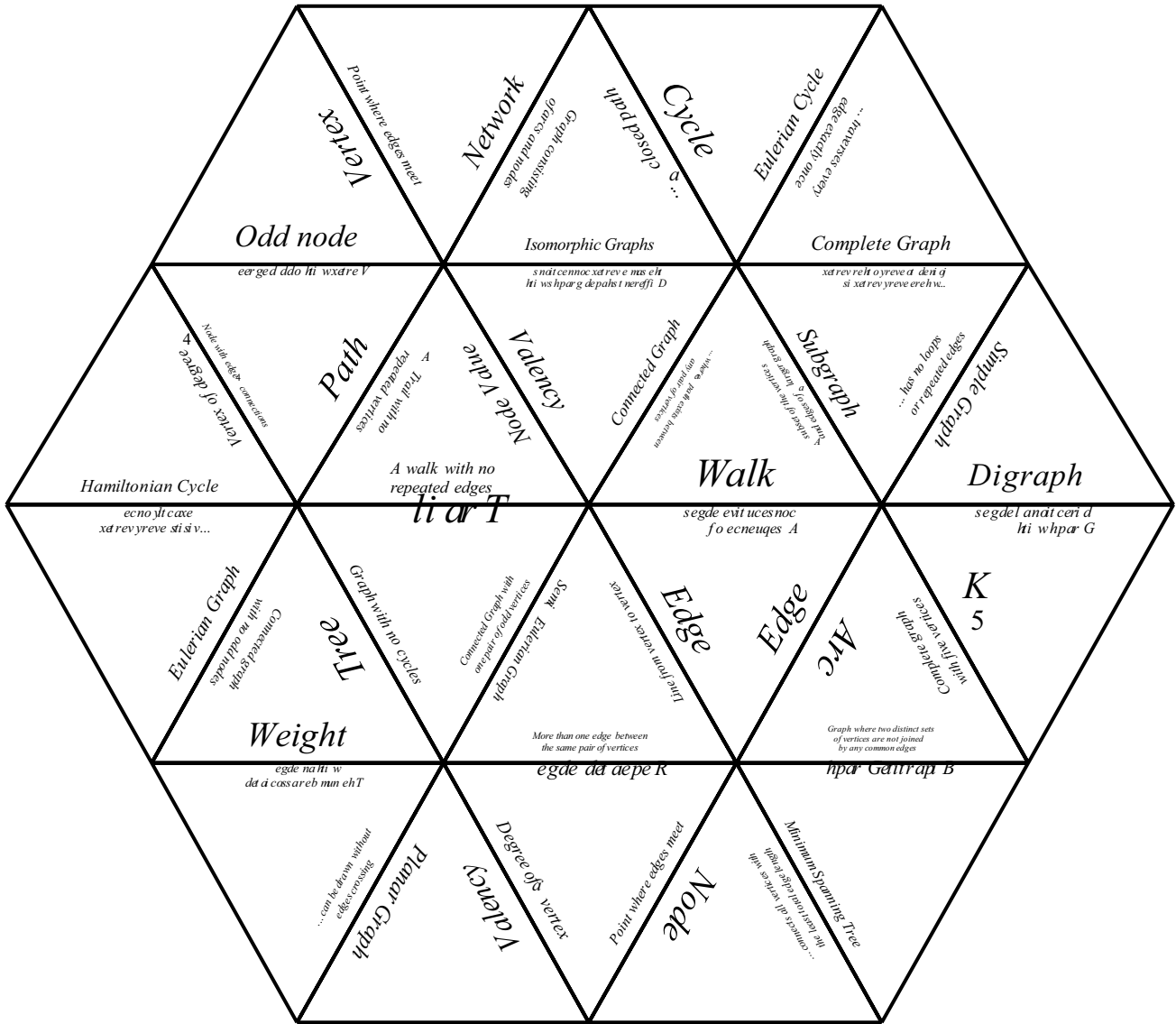
*Complete graph with five vertices*

*Graph where two distinct sets of vertices are not joined by any common edges*

*... traverses every edge exactly once*

*Arc*





## Bin Packing Exercise

Where objects of varying sizes must be placed into containers of a fixed capacity, the problem is described as 'bin packing'. The name is used for any problem of this general type, whether to do with objects, lengths, times or whatever the scenario.

Two algorithms are commonly used to attempt to solve these problems:

### First-Fit Bin Packing:

Number the bins, then always place the next item in the lowest numbered bin which can take that item.

### First-Fit Decreasing Bin Packing:

Reorder the items into decreasing order of size.

Number the bins, then always place the next item in the lowest numbered bin which can take that item.

### Activity

A builder uses piping of standard length 12 metres.

The following sections of varying lengths are required for a particular job:

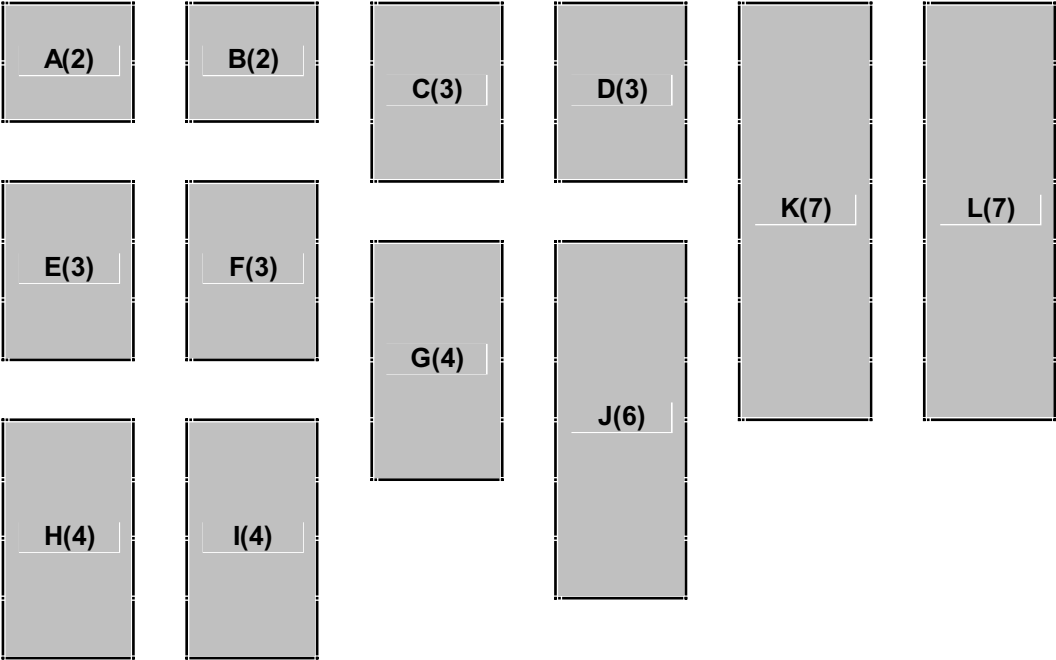
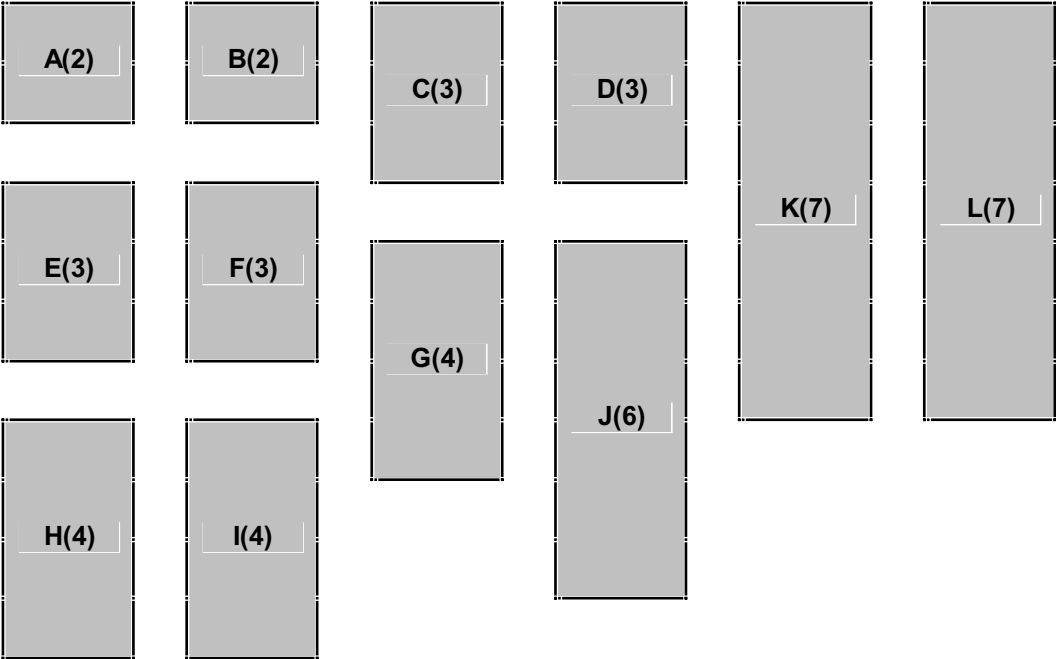
Section	A	B	C	D	E	F	G	H	I	J	K	L
Length (metres)	2	2	3	3	3	3	4	4	4	6	7	7

Explore how many standard 12 m lengths of pipe will be required if each of the following methods is used:

- (a) First-fit bin packing
- (b) first-fit decreasing bin packing
- (c) trial and improvement

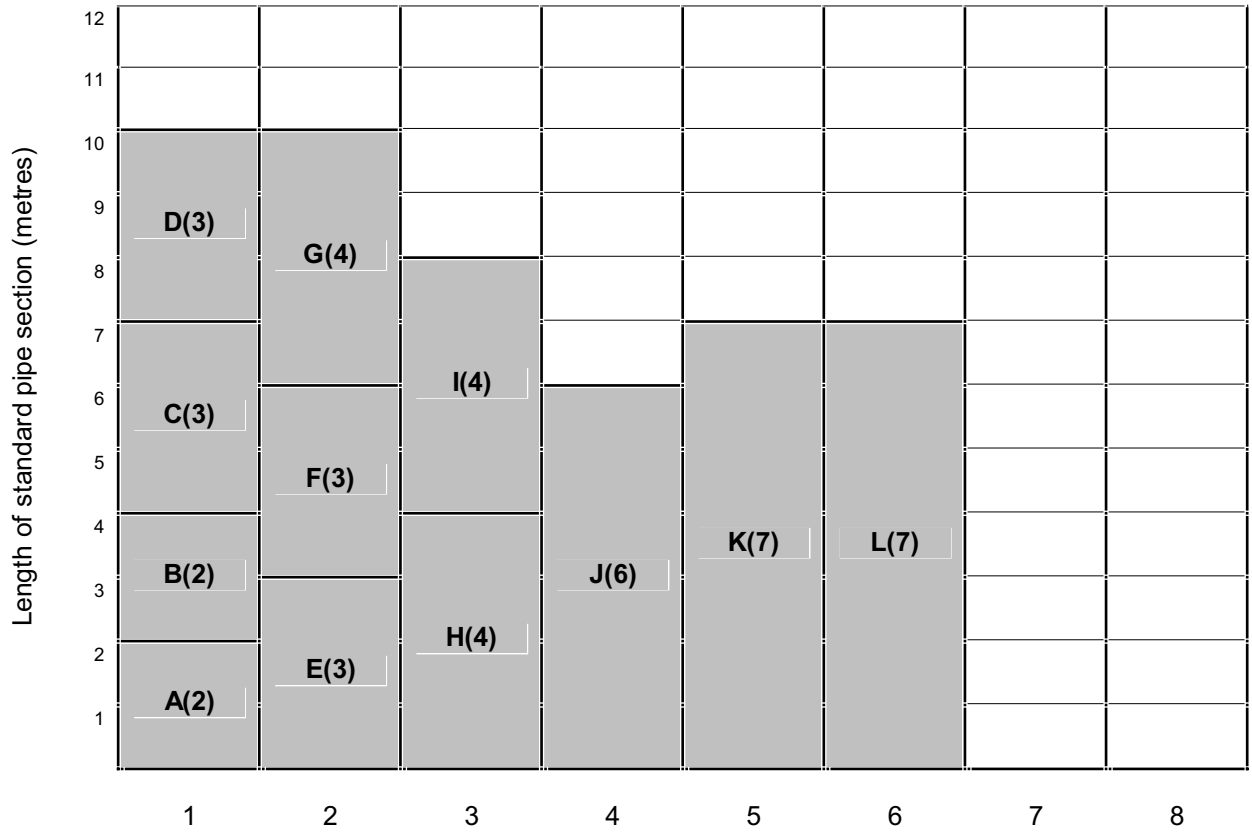


Pipe Sections

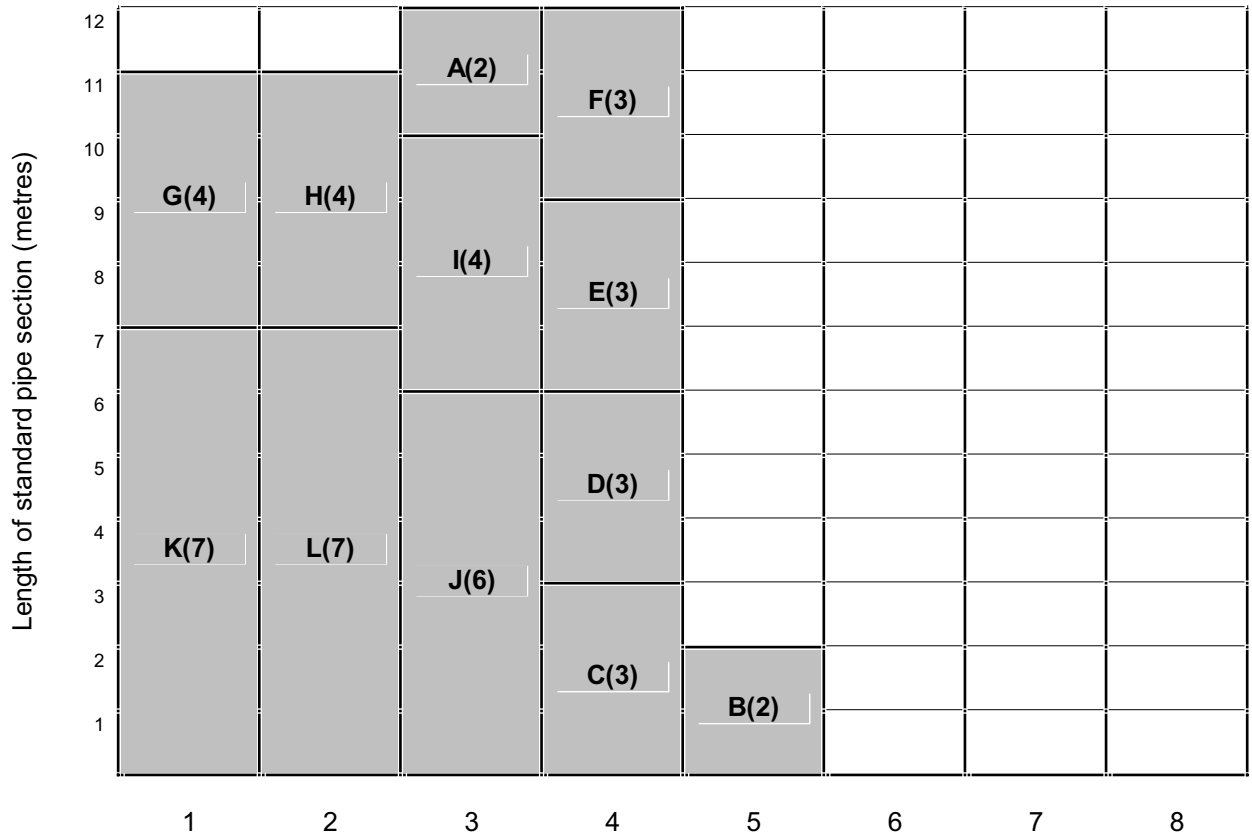


# Solutions

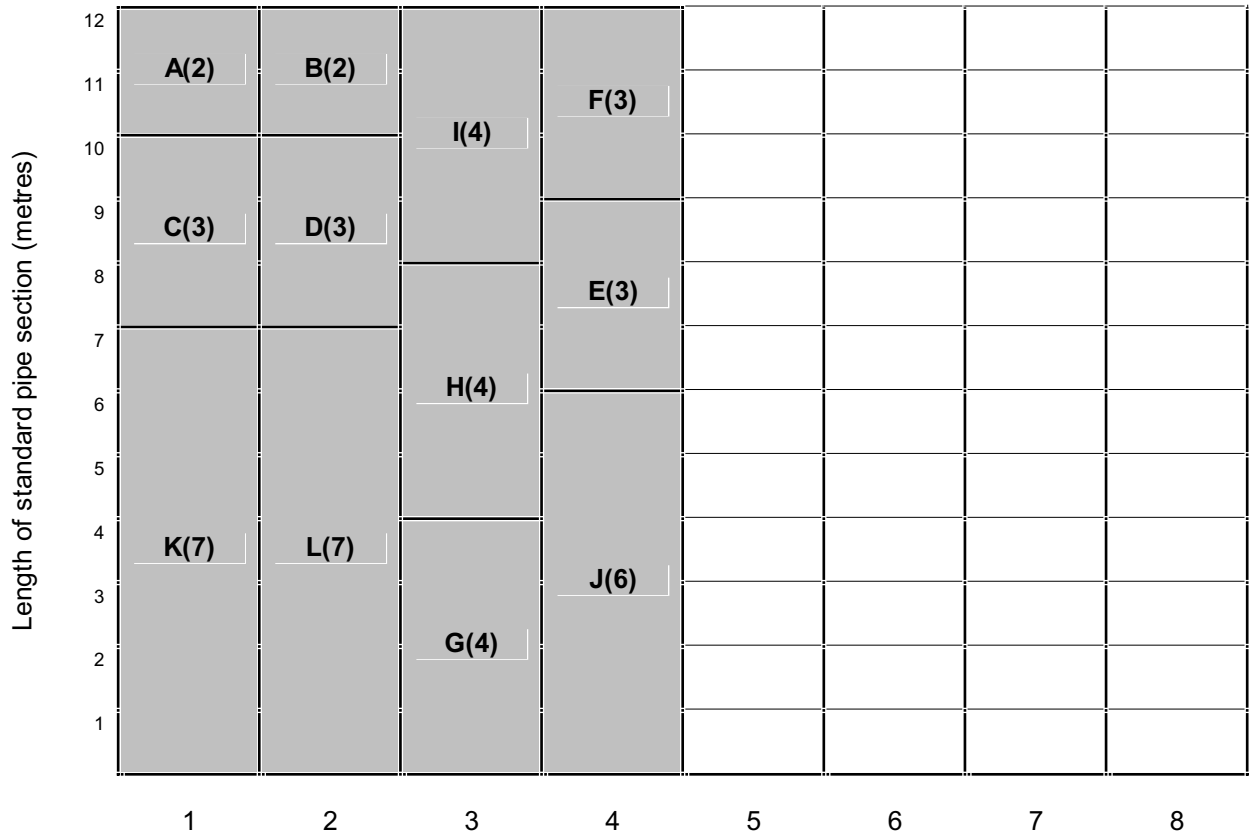
(a) First-fit bin packing



(b) First-fit decreasing bin packing



(c) Trial and improvement



# Sprouts

This is a simple pen-and-paper game for two players which involves arcs and nodes. It was invented in 1967 by Professor John H Conway and Michael S Paterson and has been described and analysed in books and on the internet. (Enter the words sprouts and Conway into a search engine for more information.)

## Sprouts with three spots:

Draw three spots anywhere on the paper.

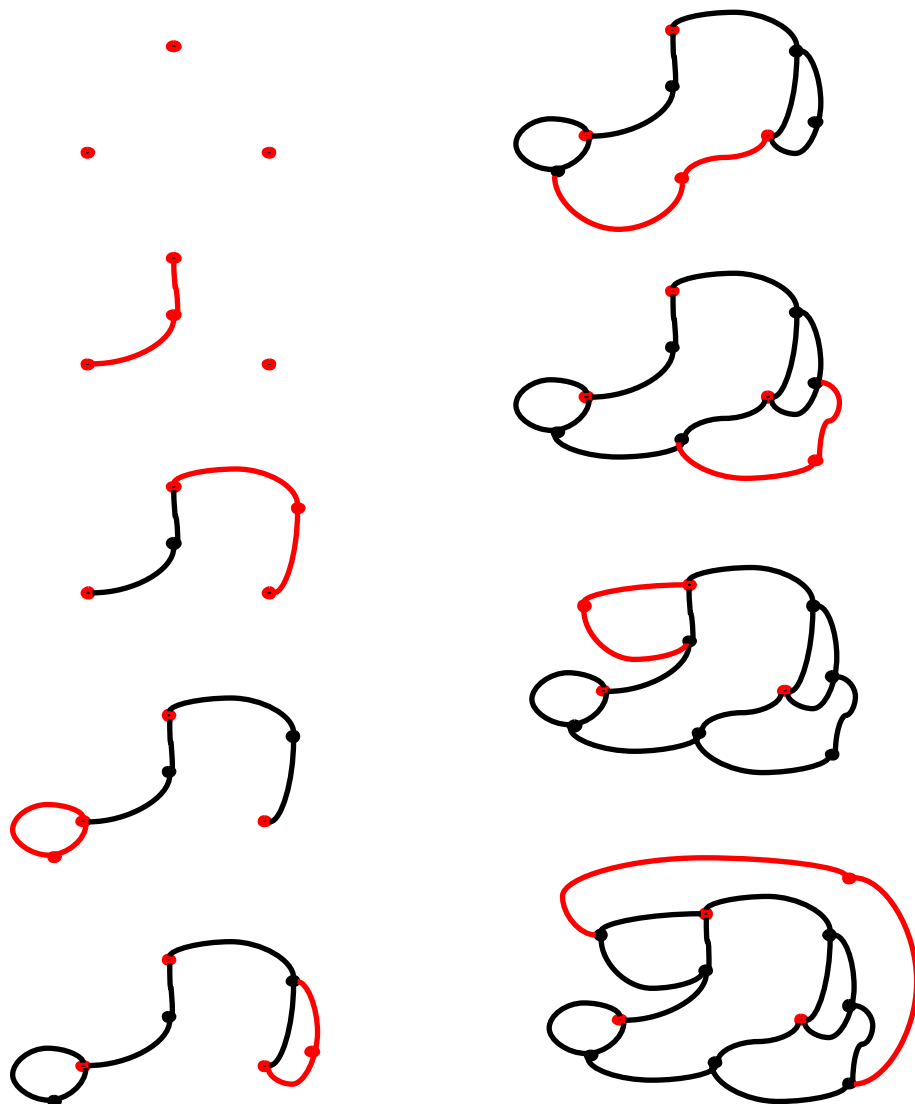
Each player in turn draws a line joining one spot to another spot (or itself) and places a new spot somewhere on this line.

No lines may cross,

No spot may have more than three lines coming out of it (i.e. we would say that the degree of any vertex cannot exceed three).

The game continues until no further moves are possible and the player to make the final move is the winner.

## A possible three spot game:



## Analysis:

Providing students have met the fact that for any graph the node-sum is twice the number of edges (from the hand-shake lemma), then it is quite easy to prove that a three spot game of sprouts can never exceed 8 moves.

The game begins with 3 vertices and no edges.

Every move adds to the network 2 edges and one vertex.

Therefore after  $m$  moves, the network will have gained  $2m$  edges and  $m$  vertices.

So the final graph contains  $2m$  edges and  $m + 3$  vertices.

Since the winning move will leave at least one vertex of degree 2, the maximum number of 3-nodes is  $m + 2$ .

Therefore  $\text{node sum} = (m + 2) \times 3 + 2$

and since  $\text{node sum} = 2 \times (\text{no. of edges})$

we have  $2 \times (2m) = (m + 2) \times 3 + 2$

so  $4m = 3m + 8$

and so  $m = 8$

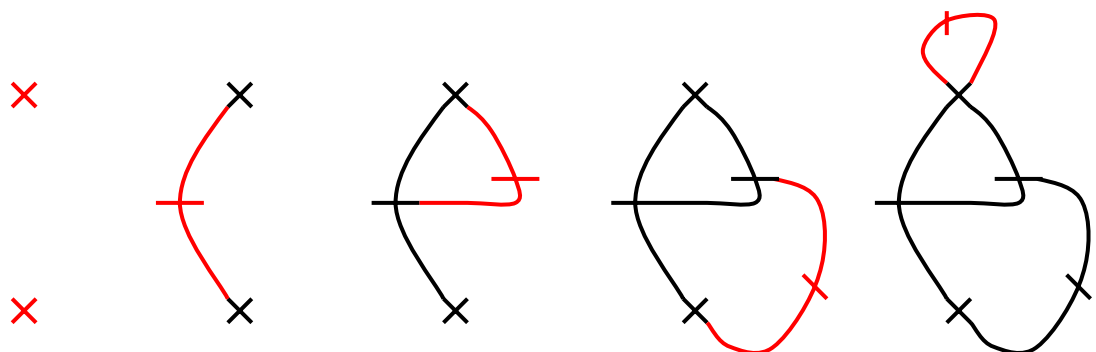
Thus no game of three spot sprouts can exceed 8 moves.

Sprouts can be played with any number of spots and it can be shown that an  $n$  spot game will never exceed  $3n - 1$  moves.

## Brussels Sprouts:

A similar game can be played with crosses instead of spots, where each cross represents a vertex of maximum degree 4. A move joins two 'branches' of existing crosses (or a cross to itself) and a cross is placed on the new line.

**The start of a possible two cross game:**



In fact in Brussels Sprouts the first player always wins if the number of crosses is odd and the second player always wins where the number of crosses is even!