Further Mathematics Support Programme

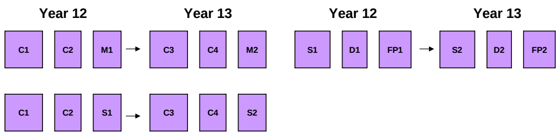
www.furthermaths.org.uk

Engaging weaker students in Decision Maths

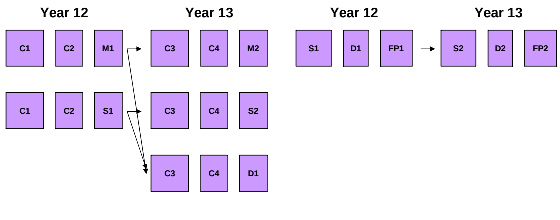
Jeff Trim

Let Maths take you Further…

Decision Maths in the KS5 Curriculum



Decision Maths in the KS5 Curriculum



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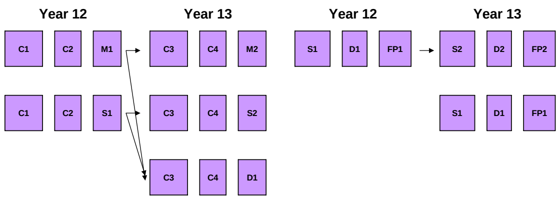

Graph Theory in Context

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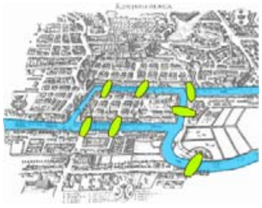
Königsberg bridges



The **Königsberg bridges** is a famous mathematics problem inspired by an actual place and situation.
The city of Königsberg on the River Pregel in Prussia includes two large islands which were connected to each other and the mainland by seven bridges. The citizens of Königsberg allegedly walked about on Sundays trying to find a route that crossed each bridge exactly once, and returned to the starting point.

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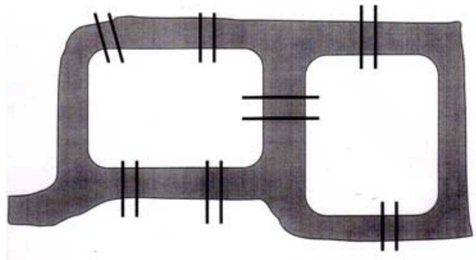
Königsberg bridges



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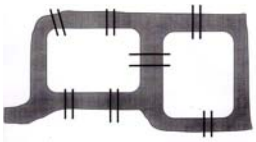
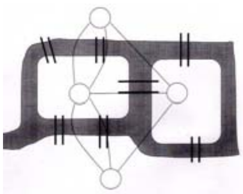
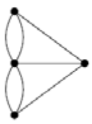
Königsberg bridges

Simplify the problem

Model it as a graph where the edges represent the bridges and the vertices represent the islands.







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Königsberg bridges

In 1736 Leonhard Euler proved that it was not possible because all the vertices of the graph are odd.

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Update: Kaliningrad

Two of the seven original bridges were destroyed during World War II. Two others were later demolished and replaced by a modern motorway.

The three other bridges remain, although only two of them are from Euler's time (one was rebuilt in 1935).

Hence there are now only 5 bridges in Konigsberg.

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Party Greetings

As the guests arrive at a party, they each shake hands with everyone present.

If five people are present, how many handshakes are there?

If ten people are present …….?

If n people are present ……….?

Party Greetings

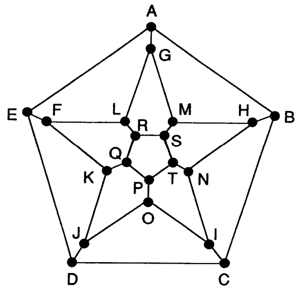
This is an introduction to the Handshake Lemma.

It also leads into Complete Graphs and using the node-sum.

Activity: The Icosian Game

(a.k.a. A Voyage Round the World)

- Find a closed cycle that visits every vertex exactly once.



Activity: The Icosian Game

(a.k.a. A Voyage Round the World)

- Two of the very few remaining original sets of Sir William Rowan Hamilton's game.

http://puzzlemuseum.com/month/picm02/200207icosian.htm

Cycles

- An **Eulerian cycle** travels along every edge in a network and returns to the starting point.
- A **Hamiltonian cycle** visits every vertex in a network and returns to the starting point.

Cycles

- A closed path is called a cycle. The sequence of vertices visited begins and ends at the same vertex.
- A **Hamiltonian** cycle is a closed path which visits each vertex once and only once.
- An **Eulerian** cycle is a closed path that travels along every edge once

Learning Styles: V.A.K.

Other Possible Activities

1. Sorts of Sorts for the Kinesthetic Learner
2. 'Imagineering' for the Visual Learner
3. Schlegal Diagrams (Isomorphic Equivalence)
4. Euler's Relation (Pattern Spotting)
5. Platonic Solids for the Artists and Historians!!

Euler's Relation

Name	Faces	Vertices	Edges
Cube			
Tetrahedron			
Octahedron			
Dodecahedron			
Icosahedron			

Euler's Relation

Name	Faces	Vertices	Edges
Cube	6	8	12
Tetrahedron	4	4	6
Octahedron	8	6	12
Dodecahedron	12	20	30
Icosahedron	20	12	30

Euler's Relation

$F + V = E + 2$

Name	Faces	Vertices	Edges
Cube	6	8	12
Tetrahedron	4	4	6
Octahedron	8	6	12
Dodecahedron	12	20	30
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Duality in front of your eyes!

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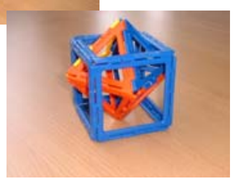
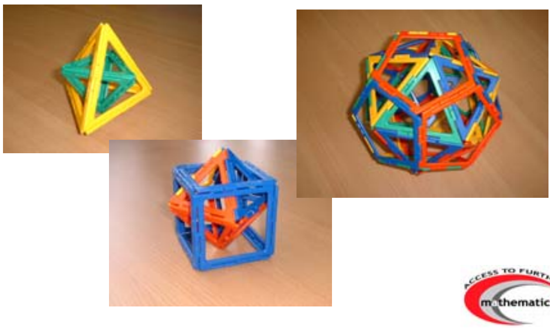


Duality in front of your eyes!



Algorithms for Sorting

Bubble Sort

Step 1: Compare the first two numbers.

Step 2: If the first number is larger than the second, exchange the numbers.

Step 3: Repeat steps 1 and 2 for all pairs of numbers until you reach the end of the list.

Step 4: Repeat steps 1 to 3 until no more exchanges are made.

Shuttle Sort

The Shuttle Sort works by comparing pairs of numbers and exchanging them if necessary.

Step 1: Compare the first two numbers and exchange if necessary.

Step 2: Compare the second and third numbers and exchange if necessary,
then compare the second and first numbers and exchange if necessary.

Step 3: Compare the third and fourth numbers and exchange if necessary,
compare the second and third numbers and exchange if necessary,
compare the second and first numbers and exchange if necessary.

Step 4: For a list of length n, continue until $n-1$ passes have been performed.

Shell Sort

The shell Sort differs from the Bubble and Shuttle methods as it compares, and possibly exchanges, non-adjacent elements.

The set of elements is split into subsets. The number of subsets for the first pass is INT(n/2), that is, the number of elements, divided by two and ignoring any remainder.

Quick Sort

Step 1: Choose any number x from the list L,
(usually the number at the mid-point – but **for AQA, choose the FIRST**).

Step 2: Write all the numbers smaller than x to the left of x, reading the original list from left to right. These form a new list L_1. Write all of the numbers larger than x to the right of x reading the original list. These numbers form a new list L_2.

Step 3: Apply steps 1 and 2 to each separate list until all of the lists contain only one number.

Step 4: The original list is now in ascending order.