

Practical Activities in Mechanics

Ted Graham
Centre for Teaching Mathematics
University of Plymouth



Friction Demonstrations



Right way up.

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Friction Demonstrations



Button side down!

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Friction Demonstrations



Press the hands together gently. With a small reaction force the book can easily be pulled down as there is little friction.

Press the hands together as hard as possible. With a large reaction force the book cannot easily be pulled down as there is a lot of friction.

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The Friction Law

$$F \leq \mu R$$

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A Calculator on a Slope

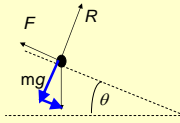


What is the coefficient of friction between the calculator and the slope?

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Solution



Parallel to the slope

$$F = mg \sin \theta$$

Perpendicular to the slope

$$R = mg \cos \theta$$

Use the friction inequality

$$F \leq \mu R$$

$$mg \sin \theta \leq \mu mg \cos \theta$$

$$\mu \geq \frac{mg \sin \theta}{mg \cos \theta}$$

$$\mu \geq \tan \theta$$

Unknown Hanging Masses

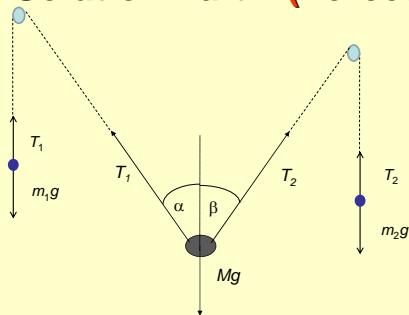


What are the unknown masses.

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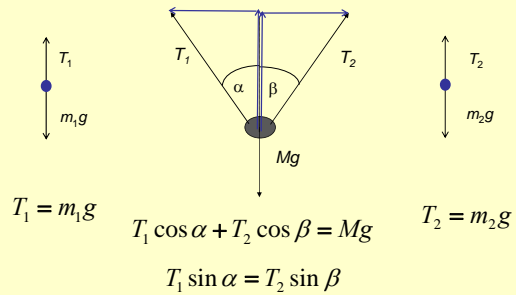
Solution Part 1 (Forces)



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Solution Part 1 (Equations)



$$T_1 = m_1 g$$

$$T_1 \cos \alpha + T_2 \cos \beta = Mg$$

$$T_2 = m_2 g$$

$$T_1 \sin \alpha = T_2 \sin \beta$$

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Solution Part 3 (Algebra)

$$T_1 \sin \alpha = T_2 \sin \beta$$

$$m_1 g \sin \alpha = m_2 g \sin \beta$$

$$m_1 = \frac{m_2 \sin \beta}{\sin \alpha}$$

$$T_1 \cos \alpha + T_2 \cos \beta = Mg$$

$$m_1 \cos \alpha + m_2 \cos \beta = M$$

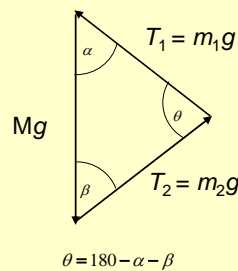
$$\frac{m_2 \sin \beta}{\sin \alpha} \cos \alpha + m_2 \cos \beta = M$$

$$m_2 \left(\frac{\sin \beta}{\sin \alpha} \cos \alpha + \cos \beta \right) = M$$

$$m_2 = \frac{M}{\left(\frac{\sin \beta}{\sin \alpha} \cos \alpha + \cos \beta \right)}$$

$$m_1 = \frac{m_2 \sin \beta}{\sin \alpha} = \frac{M \sin \beta}{\sin \beta \cos \alpha + \sin \alpha \cos \beta} = \frac{M \sin \beta}{\sin(\alpha + \beta)}$$

Alternative Solution



$$\theta = 180 - \alpha - \beta$$

$$\frac{m_1 g}{\sin \beta} = \frac{Mg}{\sin \theta}$$

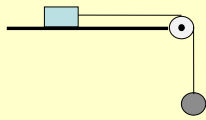
$$m_1 = \frac{M \sin \beta}{\sin \theta}$$

$$\frac{m_2 g}{\sin \alpha} = \frac{Mg}{\sin \theta}$$

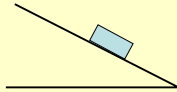
$$m_2 = \frac{M \sin \alpha}{\sin \theta}$$

Coefficient and Angle of Friction

First determine the coefficient of friction.



Then predict the greatest angle for which the block will remain at rest on the slope.



Test your prediction.

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Coefficient and Angle of Friction

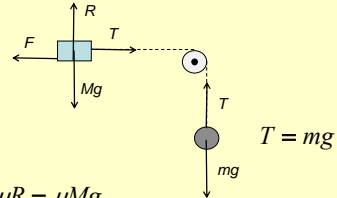
$$T = F$$

$$R = Mg$$

$$T = F$$

$$mg = \mu R = \mu Mg$$

$$\mu = \frac{m}{M}$$



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Air Resistance Problem



Find the time taken for the cone to fall from a reasonable height to the ground.

Compare with the time to fall under gravity alone.

Determine the average resistance force acting as the object falls



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Air Resistance Problem



s

$$s = ut + \frac{1}{2}at^2$$

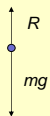
$$a = \frac{2s}{t^2}$$



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Air Resistance Problem



$$\text{Resultant Force} = mg - R$$

$$mg - R = ma$$

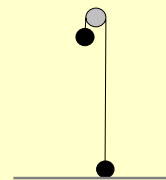
$$mg - R = \frac{2ms}{t^2}$$

$$R = mg - \frac{2ms}{t^2}$$

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Vertical Connected Particles

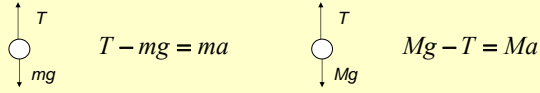


Set up the apparatus as shown in the diagram. Use a difference of 10 grams. Predict the time for the lower mass to rise to level of the pulley. Confirm with an experiment.

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Vertical Connected Particles



$$T - mg = ma$$

$$Mg - T = Ma$$

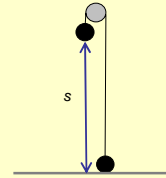
$$Mg - mg = Ma + ma$$

$$a = \frac{(M - m)g}{M + m}$$

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Vertical Connected Particles



$$a = \frac{(M - m)g}{M + m}$$

$$s = ut + \frac{1}{2}at^2$$

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Conical Pendulum Practical



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Conical Pendulum Practical



Set up a conical pendulum with the mass close to the floor.

Measure:

- Diameter of circle
- Height of suspension point
- Time for 10 circles

Calculate the angular speed in two different ways and compare.

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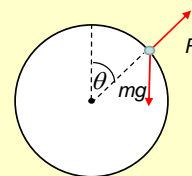
A Coin on a Cylinder



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A Coin on a Cylinder



Assume $R = 0$

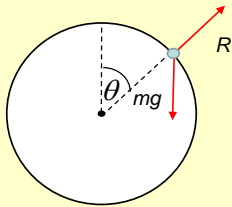
$$mg \cos \theta = m \frac{v^2}{r}$$

$$\cos \theta = \frac{v^2}{gr}$$

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A Coin on a Cylinder



$$mgr(1 - \cos \theta) = \frac{1}{2}mv^2$$

$$v^2 = 2gr(1 - \cos \theta)$$

$$\cos \theta = \frac{v^2}{2gr} = 2(1 - \cos \theta)$$

$$\cos \theta = \frac{2}{3} \Rightarrow \theta = 48^\circ$$

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A Coin on a Cylinder



Start the coin part way round the cylinder as shown in the photo.

Make a prediction and test your result.

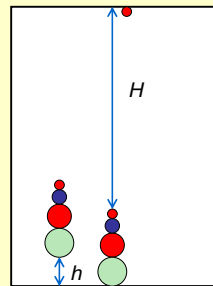
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Energy Lost?



Energy Lost?



Drop the whole thing so that the small ball just reaches a particular height, for example the ceiling.

Find the energy lost during the bounce.

Determining the Mass of a Metre Rule

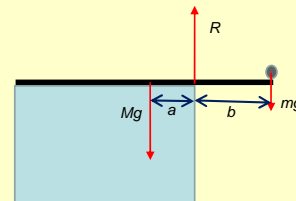


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Determining the Mass of a Metre Rule

If on the point of toppling:



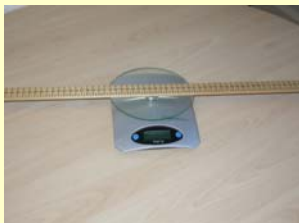
$$Mg \times a = mg \times b$$

$$M = \frac{mb}{a}$$

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Determining the Mass of a Metre Rule



Checking the prediction.

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The Suspended Beam



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Perpendicular Distances



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Perpendicular Distances



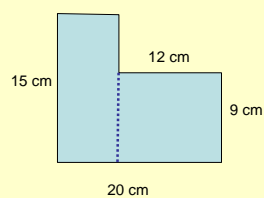
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Balance a Coke Can



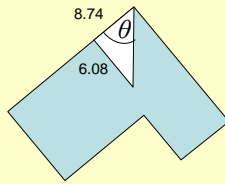
The Centre of Mass of a Lamina



$$\bar{x} = \frac{120 \times 4 + 108 \times 14}{120 + 108} = \frac{166}{19} = 8.74$$

$$\bar{y} = \frac{120 \times 7.5 + 108 \times 4.5}{120 + 108} = \frac{231}{38} = 6.08$$

The Centre of Mass of a Lamina



$$\tan \theta = \frac{6.08}{8.74}$$
$$\theta = 34.8^\circ$$

Centre of Mass of a Lamina



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