Algebra and functions

- expanding and factorising
- surds
- rationalising
- laws of indices
- dividing a polynomial
- factor theorem
- remainder theorem

Sine and cosine rule

- using both to find missing sides and angles
- finding two solutions for missing angles
- area of a triangle =½ ab sinθ

Quadratic functions

- graphs of quadratic functions
- solving equations by factorising
- completing the square
- use of the formulae
- sketching graphs of quadratic formulae

Exponentials and logarithms

- function $y=a^x$
- writing expressions as logs
- calculating using logs to base 10
- laws of logs
- solving equations of the form $a^x=b$
- changing base of logs

Equations and inequalities

- simultaneous equations by elimination
- simultaneous equations by substitution
- linear and quadratic simultaneous equations
- linear equalities
- quadratic unequalities

Binomial Expansion

- Pascal's triangle
- Combinations and factorial rule
- using $\binom{n}{r}$ in the binomial; expansion
- expanding $(a+bx)^n$

Sketching curves

- cubic functions
- reciprocal graphs and functions
- intersection points of graphs to solve equations
- transformations f(x+a), f(x-a), f(ax), af(x)

Radian measure and graphs of trig functions

- radians to measure angles
- length of an arc
- area of a sector
- area of a segment
- graphs of sine, cos and tan
- values of functions in four quadrants
- surd values for functions
- transformations of sin, tan and cos curves

Coordinate geometry

- straight line in form ax+by+c=0
- gradient of a straight line
- $y-y_1=m(x-x_1)$
- equation of a straight line
- parallel and perpendicular
- midpoint of a line
- distance between two points on a line
- equation of a circle

Differentiation

- increasing and decreasing functions
- stationary points, max, min and inflexion
- using turning points to solve problems
- derivative of f(x) as the gradient of the tangent top the graph f(x)=y
- formula for gradient of ax^n
- expanding or simplifying functions to make them easier to differentitate
- second order derivatives
- rate of change at a particular point
- equation of tangent and or normal at a point

Sequences and series

- nth term
- recurrence
- arithmetic sequences
- arithmetic series
- sun to n of an arithmetic series
- using Σ notation
- geometric sequences
- geometric series
- sum of a geometric series
- sum to infinity of geometric series

Trig Identities

- simple trig identities
- solving simple trig equations
- solving equations in the form $\sin(n\theta+a)$, $\cos(n\theta+a)$ and $\tan(n\theta+a) = k$
- solving quadratic trig equations

Integration

- simple definite integration
- area under a curve
- area under a curve negative values
- area between a line and a curve
- the trapezium rule
- integrating x^n
- integrating simple expressions
- using the integral sign
- simplifying expressions before integrating
- finding the constant of integration

Parametric equations

- Using parametric equations
- Conversion to Cartesian
- Finding the area under a curve given by parametric equations

Functions

- mapping diagrams
- range, mapping diagrams and graphs
- composite functions
- Inverse functions

The binomial expansion

- Expanding $(a+bx)^n$
- Use of partial fractions

Exponential and log functions

- $y=a^x$
- $y=e^x$
- Using e^x and the inverse of the exponential function $\log_e x$

Differentiation

- Parametric differentiation
- Implicit relations
- a^x
- Rates of change
- Simple differential equations

Numerical methods

- finding approximate roots of f(x) = 0 graphically
- Iterative and algebraic methods to find approximate roots of f(x)=0

Vectors

- Diagrams
- Unit vectors
- 2D & 3D
- Scalar product
- Vector equation of a straight line
- Angle between two lines

Transforming graphs of functions

- Modulus function |f(x)| and f(|x|)
- Solving equations involving a modulus
- Applying combinations of transformations to curves

Integration

- Standard functions
- Reverse chain rule
- Trgi identities
- Partial fractions
- Substitution
- By parts
- Numerical
- Areas and volumes
- Differential equations

Trigonometry

- functions of secant, cosecant and cotangent
- graphs of secant, cosecant and cotangent
- Simplifying expressions, proving identities and solving equations using sec, cosec and cot
- using inverse trig functions and their graphs

Partial fractions	Algebraic fractions
Further trig identities • Addition trig formulae • double angle formulae • solving equations and proving identities using double angle • using a cosθ+bsinθ • factor formula	Differentiation • chain rule • product rule • quotient rule • exponential function • logarithmic function • differentiating sinx, cos x, tanx • further trig functions

Functions challenge

The function f(x) = 2x + 4 can be written using a function machine like this:

You can reverse this by undoing the operations like this. This is called finding the inverse.

We say the inverse function is $f^{-1}(x) = \frac{x-4}{2}$

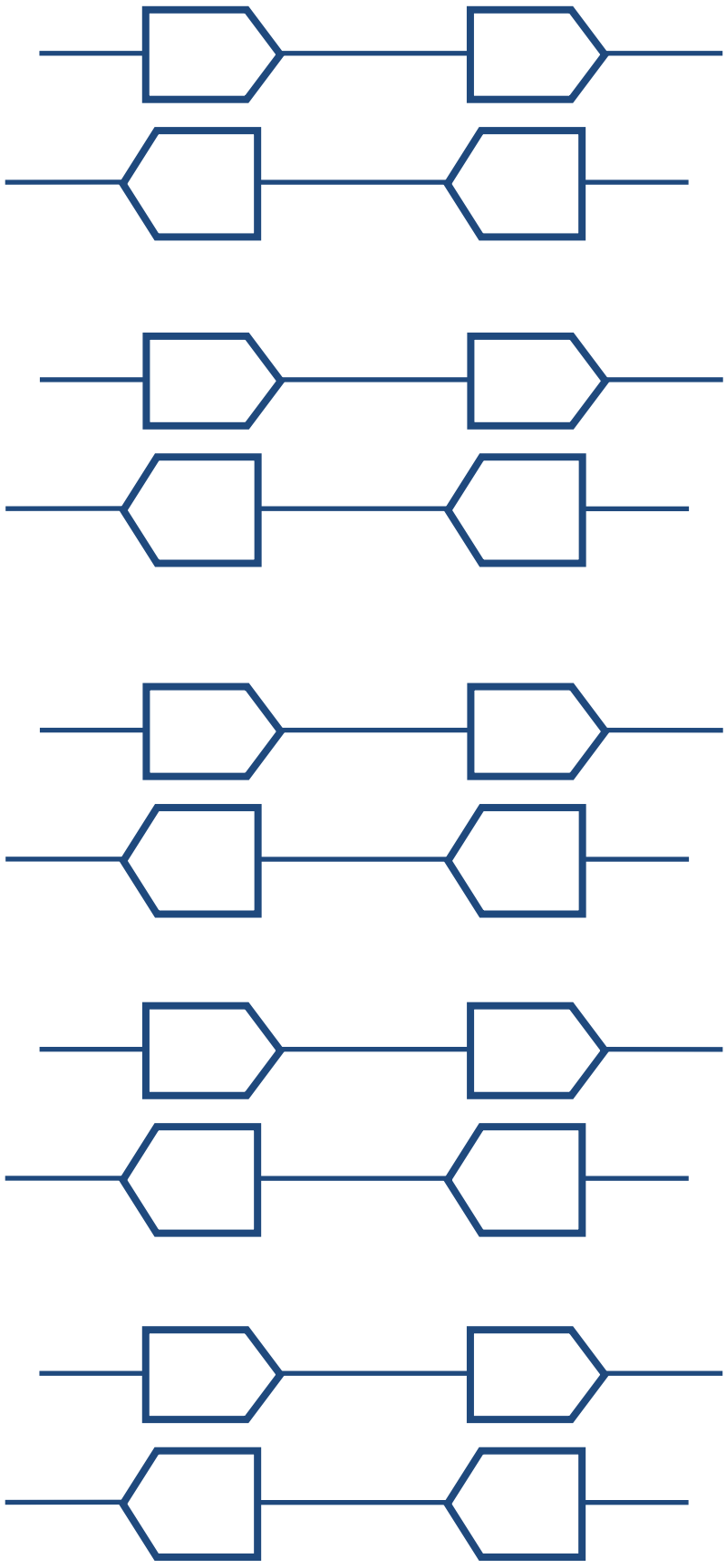
Can you find the inverses of these functions? Use the function machines if you want.

1. f(x) = 3x+5
2. f(x) = 4x+7
3. f(x) = $\frac{x}{2} + 1$
4. f(x) = $\frac{x+2}{3}$
5. f(x) = ⅔x + 3
6. f(x) = 3 – 2x
7. f(x) = x^2
8. f(x) = sin x

Draw a graph of a function and its inverse – on the same pair of axes – what do you notice?

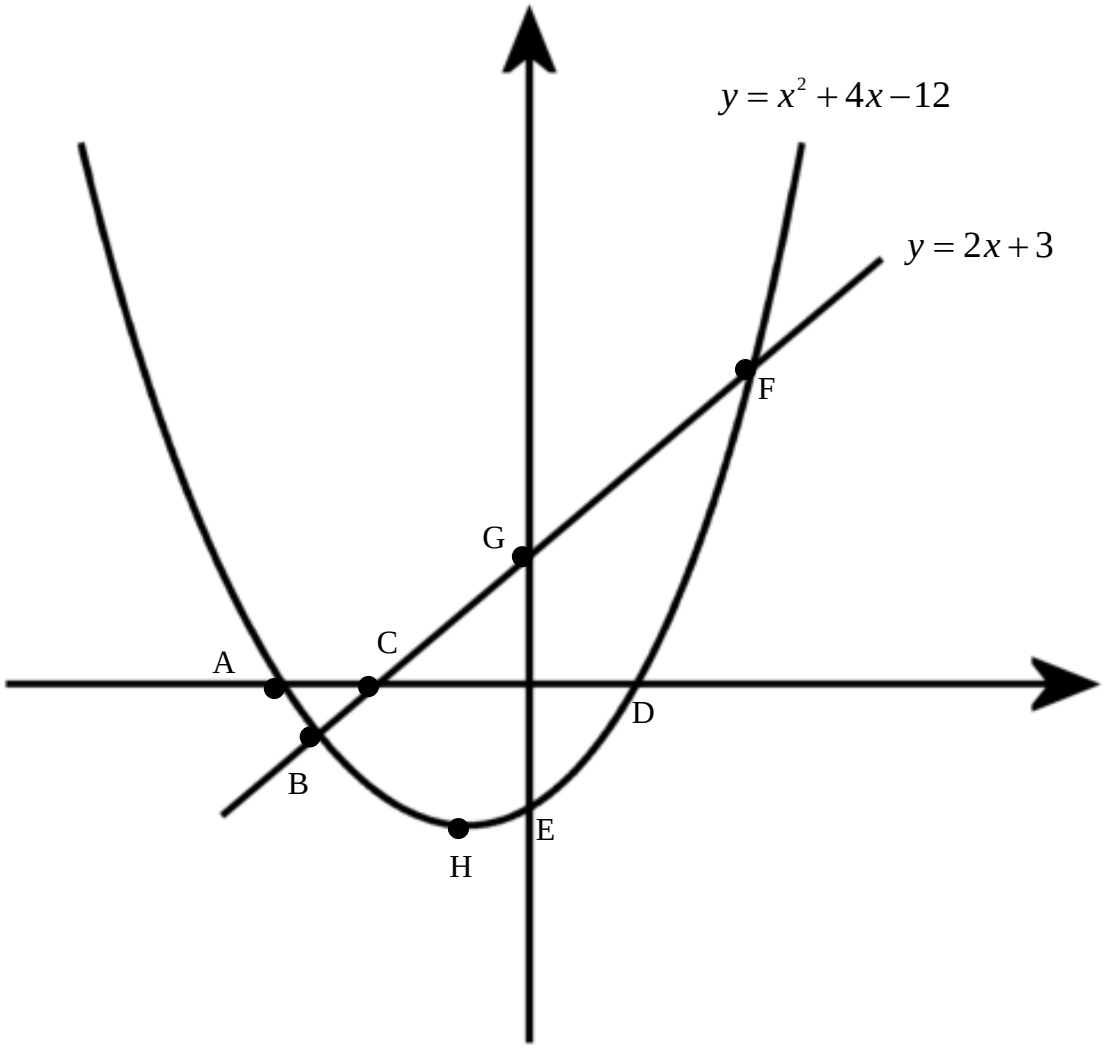
What is different about numbers 7 and 8 – why do you think this happens?

Function machines



Quadratic Challenge I

Can you work out the coordinates of all the points named with letters below?

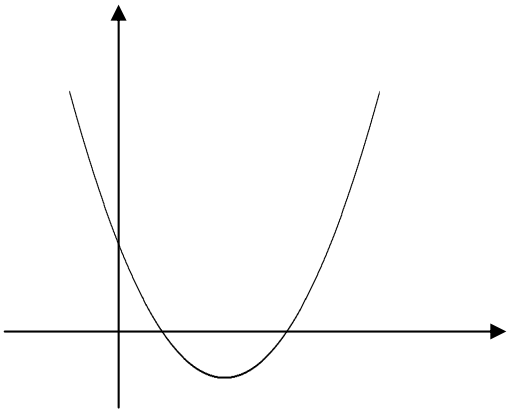


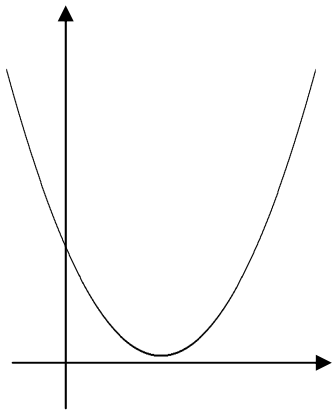
Quadratic Challenge II

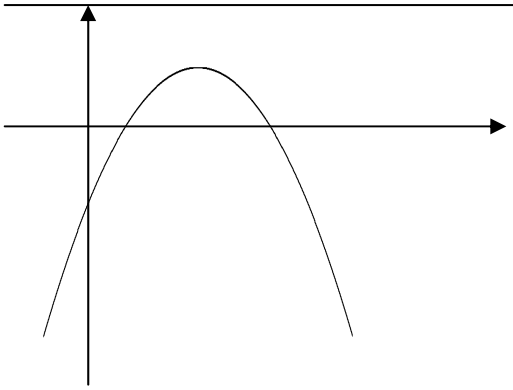
Cut out all the cards. Can you match all the cards?

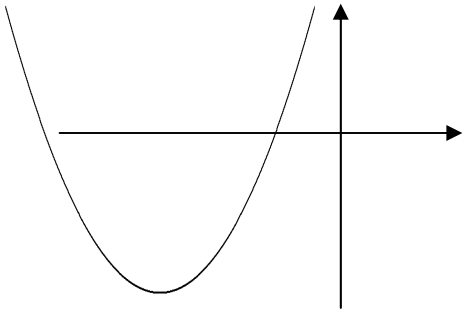
$y=x^2+6x-16$	$y=x^2-8x+16$
$y=8-x^2+2x$	$y=6x-x^2-8$
$y=x^2-10x+16$	$y=x^2+6x+8$
$y=x^2-6x-16$	$y=(x-8)(x+2)$
$y=(x+4)(x+2)$	$y=(x+2)(4-x)$
$y=(x-4)(2-x)$	$y=(x-8)(x-2)$
$y=(x-4)(x-4)$	$y=(x+8)(x-2)$
$y=(x+3)^2-25$	$y=(x-4)^2$
$y=(x-5)^2-9$	$y=-(x-3)^2+1$
$y=-(x-1)^2+9$	$y=(x+3)^2-1$

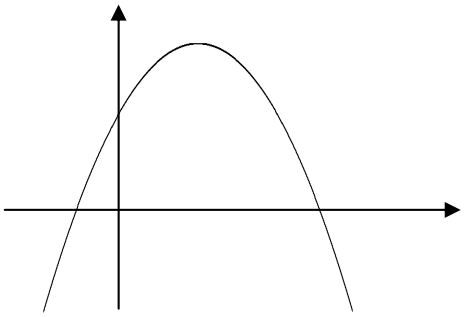
$y = (x-3)^2 - 25$	**Minimum at (3, -25)**
Minimum at (-3, -1)	**Maximum at (1, 9)**
Maximum at (3, 1)	**Minimum at (5, -9)**
Minimum at (4, 0)	**Minimum at (-3, -25)**
$x = 0,\ y = -16$	$x = 0,\ y = 16$
$x = 0,\ y = 16$	$x = 0,\ y = -8$
$x = 0,\ y = 8$	$x = 0,\ y = 8$
$x = 0,\ y = -16$	$y = 0,\ x = 8 \text{ or } -2$
$y = 0,\ x = -4 \text{ or } -2$	$y = 0,\ x = -2 \text{ or } 4$
$y = 0,\ x = 4 \text{ or } 2$	$y = 0,\ x = 8 \text{ or } 2$
$y = 0,\ x = 4$	$y = 0,\ x = -8 \text{ or } 2$

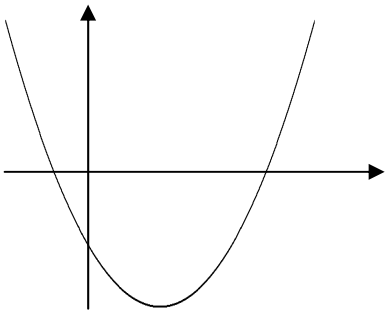


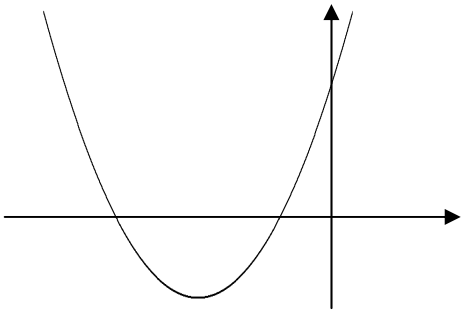












Simultaneous Equations Challenge

You can now solve pairs of simultaneous equations. What happens if there are more than two? Like these:

$$3x+4y+z=3$$

$$x+y+z=2$$

$$2x+y-z=2$$

The solution to this problem is on worksheet A but it is in the wrong order. Can you sort it out so that it is in the right order? Then try worksheet B which uses a different method.

Which method seems to work best for you?

Now try these – can you solve them? Use one of the two methods that you explored on sheets A and B.

$$x-y+zx=10$$

$$3x+y+2z=34$$

$$-5x+2y-z=-14$$

$$x+y+2z=11$$

$$2x-3y-z=-9$$

$$2x-y+3z=7$$

Type the equations above into a graph plotting programme like Autograph or Omnigraph.

- What can you see?
- Is it what you expected?
- Why is there only one solution to the set of equations?
- Can you think of a situation where there might be a different number of solutions?
- Can you describe a situation when there are no solutions – what would it look like?

Worksheet A

$3x+4y+z=3$ – equation 1

$x+y+z=2$ – equation 2

$2x+y-z=2$ – equation 3

Solve to get

$x=2$ and $y=-1$

Take away equation 2 from equation 1

$$\begin{array}{rl} & 2x+y-z=2 \\ + & x+y+z=2 \\ \rightarrow & 3x+2y=4 \end{array}$$

You now have a pair of simultaneous equations and you know how to solve these.

$2x+3y=1$ & $3x+2y=4$

Add equation 3 to equation 2

Solution is x=2, y=-1 and z=1

$$\begin{array}{rl} & 3x2 + 4x-1 + z = 3 \\ & 6 + -4 + z = 3 \\ & z = 1 \end{array}$$

Pick any of the three equations – for example equation 1 : $3x+4y+z=3$

Substitute in the values of x and y

$$\begin{array}{rl} & 3x+4y+z=3 \\ - & x+y+z=2 \\ \rightarrow & 2x+3y=1 \end{array}$$

Worksheet B

$3x+4y+z=3$ – equation 1

$x+y+z=2$ – equation 2

$2x+y-z=2$ – equation 3

Now substitute into equation 2

$x+y+z=2$ – equation 2

$x+y+(3-3x-4y)=2$

$-2x-3y+3=2$

$-2x-3y=-1$

We know

$z = 3 - 3x - 4y$

Put in values of x=2 and y=-1 and get z=1

$3x+4y+z=3$ – equation 1

$z = 3 - 3x - 4y$

Solution is x=2, y=-1 and z=1

Solve the two new equations you have made

-2x-3y=-1 & x+y=1 to get

x=2 and y= -1

Now substitute into equation 2x+y-z=2 – equation 3

$2x+y-(3-3x-4y)=2$

$5x+5y-3=2$

$5x+5y=5$

$x+y=1$

Take equation 1 and rearrange it to make z the subject of the formula