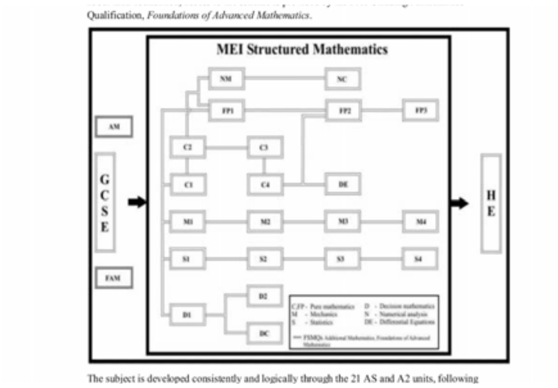
MEI

DIFFERENTIAL EQUATIONS



ASSUMED KNOWLEDGE

- Core 1,2,3,4
- Basic Kinematics eg $v = \frac{dx}{dt}$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

- Newton's Second Law F=ma
- Relevant knowledge of complex numbers

STRUCTURE

- Examination 80% (72 marks)

  Candidates to answer 3 out of 4 questions, each worth 24 marks

- Coursework 20% (18 marks)

  One modelling assignment involving the use of differential equations at an appropriate level of sophistication

CONTENT OVERVIEW

- Introduction to differential equations, concepts of general and particular solutions, initial and boundary conditions
- First order DEs – analytical approach
- First order DEs – approximate solutions
- Linear DEs with constant coefficients

  Auxiliary equation and CF+PI applied to first, second and higher order equations, with particular focus on second order

- Oscillations – SHM, damped HM and (as enrichment) forced HM
- Systems of Differential Equations

STYLE OF APPROACH

- Modelling tool
- Illustrated through practical examples
- Exercises contain variety of situations for applications, such as kinematics, population growth, electronics, cooling, oscillations, radioactivity, water flow…..
- Use of spreadsheet where appropriate
- Students encouraged to sketch solution curves and interpret them eg long term behaviour
- Investigations

ONLINE RESOURCES

As with all modules, each chapter has:

- ❖Study plan
- ❖Notes and examples
- ❖Crucial points
- ❖Hints and worked solutions for exercises in textbook
- ❖Glossary
- ❖Multiple choice test
- ❖Chapter assessment

(Some chapters have interactive resources.)

Note also ‘PLUS’ article!

Chapter 1: Using Differential Equations in Modelling

- Concept of DE as a rate of change introduced via modelling
- Use of modelling cycle to build up and improve model
- Ideas of general solutions and family of curves *(Autograph with constant changer particularly helpful here)* and particular solutions where boundary or initial conditions are given.
- Sketching and interpreting graphs, in particular ICs, general trends, long term behaviour etc. *(Autograph and tangent fields helpful eg cooling coffee.)*
- Process of verification for general and particular solutions

Chapter 2: Tangent Fields

- The tangent field as a field of small direction indicators *(probably familiar with them by now from using Autograph in Chapter 1)*
- Creating tangent fields manually - requiring a systematic, efficient approach
- The use of isoclines
- Interpreting particular solution curves on their tangent field and using appropriate vocabulary to describe them.

Chapter 3: Separation of Variables

- Method of separation of variables for suitable first order differential equations.
- Interactive activity in online resources
- Many situations can be modelled by separable equations with only a function of the dependent variable on the RHS (ie) eg growth/decay, Newton’s law of cooling, vertical motion against a resistance. In fact most of the questions in the exercise are like this.
- Students are expected to interpret their solutions in terms of the physical situation being modelled.
- *Some students will already have learnt the method of separation of variables in Core 4. However the extensive exercise contains plenty of questions that will allow them to consolidate their technique and practise other important skills. Some of the questions will challenge even the most competent students.*

Chapter 4: Integrating Factors

- First order linear differential equations – general form $\frac{dy}{dx} + P(x)y = Q(x)$
- Integrating factor method (based on transforming LHS to exact derivative):

  I.F.= $e^{\int P(x)\,dx}$
- Solving equations to find general and particular solutions.

Chapter 5: Euler’s Method

- “Approximate numerical solutions” – step-by-step method
- First order differential equations of the form $\frac{dy}{dx} = f(x, y)$
- Use of Euler’s method with initial conditions given and step size h

  Interesting concept – “threading through tangent field”
- By “approximating the solution curve” we mean finding approximate values of y corresponding to a set of values of x. We can then plot these to see the approximate shape of that part of the curve.

  (In this chapter, Euler’s method is usually applied to estimate the value of y for a specific value of x, a short distance from the point where the initial conditions are given. Use of speadsheets for this.)
- Error analysis – students are encouraged to use spreadsheets to investigate the effect of changing the step size. (*In general, the smaller the step size, the smaller the error, but the greater the number of steps needed the longer the process.)*
- The relation between step size and error is further explored and the results of this are used to find a better estimate more efficiently (by extrapolating a graph of estimate against step size) and to calculate the step size needed for a given degree of accuracy.
- Improved Euler met as investigation at end of chapter

Chapter 6: Linear equations with constant coefficients

- Auxiliary equations introduced for first, second and higher order equations (with RHS=0)
- General and particular solutions. *To find the values of the unknown constants in the solution of a second order DE, we may be given* initial conditions *(two pieces of information given for the same value of the independent variable) or* boundary conditions *(one piece of information for each of two values of the independent variable).*
- Focus on second order equation of the form $a\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+b\frac{\mathrm{d}y}{\mathrm{d}x}+cy=0$
- Nature of the roots of the auxiliary equation $a\lambda^2+b\lambda+c=0$ and the corresponding solutions to the DE.
- Non-homogeneous equations (RHS non-zero), CF+PI, trial functions
- General and particular solutions
- Use of Autograph to illustrate/check solution curves

Chapter 7: Oscillations

- Simple harmonic motion $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+\omega^2x=0$
- Improving the model: linear damping (dashpots)
- Damped harmonic motion modelled by

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+\alpha\frac{\mathrm{d}x}{\mathrm{d}t}+\omega^2x=0$$

- Overdamping, critical damping and under-damping

Chapter 8: Forced Oscillations

NB This is an enrichment chapter – included for completeness

- The undamped case (eg a particle on the end of a spring, the other end of which is forced to oscillate)
- Damped forced harmonic motion *(as above but with resistance eg in the form of a dashpot)*
- Resonance

Chapter 9: Systems of Differential Equations

- Systems of DEs involving 2 or more dependent variable and 1 independent variable
- Linear systems for 2 dependent variables and their solution
- Equilibrium points
- Investigation of non-linear systems:
  - Tangent fields
  - Investigating equilibrium points
  - Step-by-step methods
- Interactive resources for predator-prey

COURSEWORK

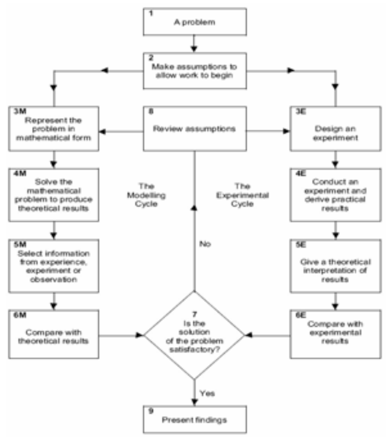
(See Differential Equations Coursework Guidance and Differential Equations Coursework Tasks in the Additional Teaching Resources in the online resources)

6-8 hours

Experimentation and modelling

Modelling Cycle

Mark scheme A (more modelling) or B (more experimentation)



Coursework Tasks

1) Cascades
2) Damped Oscillations
3) Parachutes
4) Bungy Jumping
5) Paper cake cups
6) Aeroplane landing
7) Growth functions
8) Balloon ascents
9) Interacting species
10) Oscillating spring

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Isabel Cooper – July 2007