

MEI
STRUCTURED MATHEMATICS

MEI conference
University of Reading
July 5-7 2007

C3 COURSEWORK

*What is this coursework designed to do,
how do we prepare students for it and
how do we assess it?*

Presenter: Val Hanrahan

The coursework rationale is:

The aims of this coursework are that students should appreciate the principles of numerical methods and at the same time be provided with useful equation solving techniques.

The objectives are:

- that students should be able to solve equations efficiently, to any required level of accuracy, using numerical methods;
- that in doing so they will appreciate how to use appropriate technology, such as calculators and computers, as a mathematical tool and have an awareness of its limitations;
- that they show geometrical awareness of the processes involved.

The coursework requirements are:

Candidates will investigate the solution of equations using the following three methods:

- Systematic search for a change of sign using one of the three methods: decimal search, bisection or linear interpolation.
- Fixed point iteration using the Newton-Raphson method.
- Fixed point iteration after rearranging the equation $f(x) = 0$ into the form $x = g(x)$.

NB: A different equation must be used for each method. Students should avoid:

- Any equation used in Chapter 6 of the C3/C4 textbook.
- Trivial equations. For an equation to be non-trivial it must pass two tests:
 - It should be an equation they would expect to work on rather than just write down the solution (if it exists); for instance $\frac{1}{(x-a)} = 0$ is definitely not acceptable; nor is any quadratic (since there is the quadratic formula), or any polynomial expressed as a product of linear factors.
 - Constructing a table of values for integer values of x should not, in effect, solve the equation. Thus $x^3 - 6x^2 + 11x - 6 = 0$ (roots at $x = 1, 2$ and 3) is not acceptable.

Centres **may** provide students with a list of at least ten equations from which they can, if they wish, select those they are going to use, but any list must be forwarded to the moderator with the sample and changed for each examination session. Centres may refuse to issue a list on the grounds that candidates benefit from the mathematics they learn while finding their own equations.

The coursework assessment sheet is both detailed and prescriptive and should be issued to students as they start their coursework. Students should be encouraged to number their pages and enter in the 'comment' column the page number where they think each of the criteria is satisfied. This is helpful to them, as they can check that everything is included; to you, as you can see where they think it is, if it is not immediately apparent; and also to the moderator.

Domain	Mark	Description	Comment	Mark
Change of sign method (3)	1	The method is applied successfully to find one root of an equation.		
	1	Error bounds are stated and the method is illustrated graphically.		
	1	An example is given of an equation where one of the roots cannot be found by the chosen method. There is an illustrated example of why this is the case.		

Notes:

- Linear interpolation is not generally used.
- 3 decimal places is an acceptable level of accuracy.
- For both decimal search and interval bisection, the error bounds are contained within the method. The accuracy of any root should be stated in the form of error bounds ($x = 2.614 \pm 0.0005$) or solution bounds ($2.6135 \leq x \leq 2.6145$)
- The method must be illustrated for **their** equation, showing clearly where there is a change of sign. It is not sufficient to do a general illustration.
- Failure only usually occurs when the methods are applied blindly, for example manipulating the equation without first drawing a sketch graph, but it is possible to find examples where a graph drawn to a reasonable scale only appears to have one root in a unit interval, but in fact has more.

Domain	Mark	Description	Comment	Mark
Newton-Raphson method (5)	1	The method is applied successfully to find all the roots of a second equation. All roots are found.		
	1	The method is illustrated graphically for one root.		
	1	Error bounds are established for one of the roots.		
	1	An example is given of an equation where this method fails to find a particular root, despite a starting value close to it.		
	1	There is an illustrated explanation why this has happened.		

Notes:

- This equation must be different from the first one.
- Roots should be found to at least 5 significant figures.
- The graphical illustration is for only **one root** and must relate to **their** equation.
- Error bounds must be established by looking for a change of sign and not just stated.
- Failure of the method requires that **a particular root** is not found, despite a starting value close to it. This means that either there is no root using that starting point, or the method converges to a different root from the one that was being sought.
- Again the illustration must relate to their equation.

Domain	Mark	Description	Comment	Mark
Rearranging $f(x) = 0$ in the form $x = g(x)$ (4)	1	A rearrangement is applied successfully to find a root of a third equation.		
	1	Convergence of this rearrangement to a root is demonstrated graphically and the magnitude of $g'(x)$ is discussed.		
	1	A rearrangement of the same equation is applied in a situation where the iteration fails to converge to the required root.		
	1	This failure is demonstrated graphically and the magnitude of $g'(x)$ is discussed.		

Notes:

- The magnitude of $g'(x)$ is **discussed**. It is not necessary to be able to differentiate $g(x)$, since comparing the gradient of the curve near the point where $x = g(x)$ with either a line of gradient 1 or a line of gradient -1, as appropriate, gives a very satisfactory discussion.
- For failure it is possible either to use the same rearrangement and a different root, or a different rearrangement and the same root, or a different rearrangement and a different root.

Domain	Mark	Description	Comment	Mark
Comparison of methods (3)	1	One of the equations used above is selected and the other two methods are applied successfully to find the same root.		
	1	There is a sensible comparison of the relative merits of the three methods in terms of speed of convergence.		
	1	There is a sensible comparison of the relative merits of the three methods in terms of ease of use with available hardware and software.		

Notes:

- Students should realise that their experiences are very limited, and should base any comparisons solely on these.
- They need to say what hardware and software they have been using, since any comments they make will be relative to this.

Domain	Mark	Description	Comment	Mark
Written communication (1)	1	Correct notation and terminology are used.		

Notes:

- A common error is to confuse 'function' and 'equation' by saying, for example 'I am going to use decimal search to solve the equation $f(x) = x^5 - 5x + 3$ '

Domain	Mark	Description	Mark
Oral communication (2)	2	Presentation Please tick at least one and give a brief report. Interview Discussion	

Notes:

- Each student must talk about the task. Topics for discussion may include strategies used to find suitable equations and explanations, with reference to graphical illustrations, of how the numerical methods work. An interview is also an ideal opportunity to pursue further any areas where there are errors or omissions in the coursework, although this will not affect the mark given in that domain.

Half marks may be awarded, but the overall total must be an integer so the assessor must make the decision to round up or down.

The total mark for this component is 18, so it constitutes 20% of the marks for this unit.

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*What I do to prepare my students for this
component in order to give a broad
learning experience.*

Presenter: Val Hanrahan

Preparatory work

We have tried starting the coursework in Year 12 after the examinations, but we have very little time (we have already finished for the summer) and classes are always incomplete since traditionally this is the period allotted to educational visits and field trips.

Last year