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“Support rather than illumination”
statistical misconceptions

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2 quotes:

- Most people use statistics as a drunken man uses a lamp post - for support rather than for illumination. - Andrew Lang
- A judicious man uses statistics, not to get knowledge, but to save himself from having ignorance foisted upon him. - Thomas Carlyle

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Why?

- Children with bigger feet spell better.
- Areas with higher divorce rates generally have lower death rates.
- Nations that add fluoride to their water have a higher cancer rate than those that don't.

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Possible explanations

- Children with bigger feet tend to be older so they spell better.
- Older couples are less likely to divorce. Areas with a high divorce rate have more younger couples so the death rate is lower.
- Nations that add fluoride to water are richer; their citizens live long enough to develop cancer.

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Progress from KS2 to KS3 by tier of entry at KS3

KS2 level	No of students	KS3 level						
		2	3	4	5	6	7	8
3-5								
2	34	0.0	85.3	11.8	0.0			
3	662	0.3	23.0	64.8	11.3			
4	317	0.0	4.2	52.1	43.5			
5	5	0.0	40.0	40.0	20.0			
4-6								
3	379		0.3	24.9	65.3	9.5		
4	1538		0.0	4.6	48.8	46.3		
5	85		0.0	1.2	18.8	80.0		
5-7								
3	20			0.0	25.0	70.0	5.0	
4	1236			0.2	6.6	65.9	27.2	
5	926			0.1	0.9	25.6	73.4	
6-8								
4	138				0.0	18.1	76.8	5.1
5	1024				0.0	3.0	50.9	46.1

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$P(A|B) \neq P(B|A)$

- “Most burglaries take place between midnight and 7am so if you see anyone out and about between those times, ring the police.”
Neighbourhood Watch Newsletter

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Is terrorist profiling a good idea?

- “Islamic terrorism in the West has been universally carried out by young Muslim men, usually of ethnic appearance, almost always travelling alone or in very small groups.”

News of the World

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Probability on trial

- O J Simpson’s lawyer argued that fewer than 1 in a 1000 women who are abused by their mates go on to be killed by them so the abuse in the Simpsons' marriage was irrelevant to the case.
- Is this a reasonable argument?

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Using the appropriate probability

- Nicole Simpson was killed.
- “Given certain reasonable factual assumptions, it can be easily shown using probability theory that if a man abuses his wife and she is later murdered, the batterer is the murderer more than 80% of the time.”

John Allen Paulos

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The case of the stolen handbag

- In 1964 an elderly woman in Los Angeles had her handbag stolen by a woman with a blonde ponytail who made her getaway in a yellow car driven by a black man with a moustache and a beard. The police investigation found a blonde suspect with a ponytail who lived with a black man who had a moustache, a beard and a yellow car. Nobody could identify the suspects.

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The people v Collins

Police evidence was based on probability

Characteristic	Probability
Yellow car	0.1
Man with moustache	0.25
Black man with beard	0.01
Woman with ponytail	0.1
Woman with blond hair	0.33
Interracial couple in car	0.01

The overall probability was calculated as 1 in 12 million.

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The Appeal

- There are 1 000 000 couples in Los Angeles.
- If the probability of a couple having all the characteristics is 1 in 12 million, the probability of there being no other couple with all these characteristics is...
- Based on the 8% chance of there being another such couple, the conviction was overturned.

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Prosecutor's and Defendant's Fallacies

- Suppose there is scientific evidence which only one in a thousand people would match.
- The prosecutor's fallacy says this means there is only a one in a thousand chance that the defendant is innocent.
- The defendant's fallacy says that there are 60 million people in the UK. 60 000 of them would produce this evidence so there is only a 1 in 60 000 chance of guilt.

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Bayes' theorem

$$P(A | B) = \frac{P(B | A)P(A)}{P(B | A)P(A) + P(B | A')P(A')}$$

- We want to find P(guilty|evidence).
- P(evidence|guilty)=1
- P(evidence| not guilty) = 1/1000
- We don't know P(guilty).
- Bayesians assign a prior degree of belief as this probability.
- Bayesian calculations have been tried with juries.

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If p is the degree of belief in the defendant's guilt, the evidence changes this.

$$P(\text{guilty} | \text{evidence}) = \frac{p}{p + (1 - p) \times 0.001}$$

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The power of the headline

- "Thirteen-year-olds get drunk once a week"
TES headline 10.11.06
- What does this mean? What questions does it raise?
- The story says: "One in every nine 13-year-olds gets drunk every week, a survey of more than 33, 000 children at state secondary schools has revealed."

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Found on the internet

- If you gave every human on earth an equal portion of dry land (including uninhabitable area), everyone would get a plot of land roughly 3 metres by 3 metres.
- Could this be true?

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Counting

- In the United States there are two ways of counting visually impaired children.
- American Printing House for the Blind (APH) uses the restricted definition of legal blindness of 20/200 or less.
- Individuals with Disabilities Education Act (IDEA) requires a definition that can include those children that are totally blind as well as those who are partially sighted or low vision.
- Which count will produce the higher figure?

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- In the 1994 APH census, the count for children was 53,576 (legally blind only). In 1993-94 the IDEA child count was 24,892 (includes the partially sighted).
- Why?
- One reason could be that students with visual impairment also have another disability and are classified under that category instead.
- So what?
- If statistics determine funding and the statistics are wrong ...

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Surveys and reports

- “The evidence suggests that a majority of people who use drugs are able to use them without harming themselves or others.”

Drugs – facing facts
The report of the RSA Commission
on Illegal Drugs, Communities and Public Policy
March 2007

- What does this mean?
- What is the evidence?

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The survey

- YouGov conducted 2 surveys; one of the general public and one of drug users.

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From the survey of drug users

- **Going back to the subject of harm and focusing only on illegal drugs, how much harm do you think your using drugs has done you personally? (%)**

A lot	1
A fair amount	8
Not much	50
None at all	40
Don't know	1

- **And how much harm do you think your using drugs has done your family and those around you, including your neighbours? (%)**

A lot	1
A fair amount	3
Not much	16
None at all	80
Don't know	1

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How much harm is each of the following likely to cause to individuals and their families and friends? (% saying 'a great deal' or 'some')

	Drug users	General public
Injected heroin	98	97
Crack cocaine	97	96
Smoked heroin	96	96
Solvents	92	93
'Crystal meth'	89	90
Tobacco	88	90
Alcohol	86	83
Powder cocaine	84	94
LSD	74	86
Ecstasy	68	92
Ketamine	61	n/a
Prescribed tranquillizers	54	55
Magic mushrooms	37	n/a
Cannabis	23	64
Coffee	7	10

Note: The general public survey did not ask respondents about ketamine or magic mushrooms.

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From the survey of drug users

- **Earlier we asked about the harm that different substances may possibly cause and about how addictive they may be. Now we would be grateful if you could indicate which of the following substances you yourself use, either frequently or only occasionally. (%)**

Cannabis	87
Alcohol	71
Tobacco	65
Coffee	63
Powder cocaine	31
Ecstasy	29
Magic mushrooms	18
Ketamine	7
Prescribed tranquillizers	6
LSD	6
'Crystal meth'	2
None of these	1
Solvents	1
Crack cocaine	1
Injected heroin	0
Smoked heroin	0

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Representative?

“However, the one group almost certainly underrepresented in YouGov’s sample is dependent heroin users. Their lifestyles are liable to make them hard to reach by any ordinary – and probably any extraordinary – research method.”

RSA Commission on Illegal Drugs,
Communities and Public Policy
YouGov survey results

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- “more than one in six young people leave school unable to read, write and add up properly.”

Leitch Report
Prosperity for all
in the global economy
world class skills
Dec 2006

- What skills do these young people lack?

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Skills for Life Survey, DfES, 2003.

- Around one in six were classified at Entry level 3 or below in the literacy assessment – most of these met the entry level 3 requirement.
- Nearly one in two were classified at Entry level 3 or below in the numeracy assessment, including one in five at Entry level 2 or below.

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Adding up properly?

- **Numeracy** *An adult classified at this level...*
- **Entry level 1** Understands information given by numbers and symbols in simple graphical, numerical and written material.
- **Entry level 2** Understands information given by numbers, symbols, simple diagrams and charts in graphical, numerical and written material.
- **Entry level 3** Understands information given by numbers, symbols, diagrams and charts used for different purposes and in different ways in graphical, numerical and written material.

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Things to look out for (from Darrell Huff’s “How to lie with statistics”)

- Who says so? (look out for bias – could be unconscious)
- How does he know? (is the sample large enough)
- What’s missing? (sample size, comparison, other factors)
- Did somebody change the subject? (what people say may not be the same as they do)
- Does it make sense? (watch out for extrapolation)

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You can prove anything with statistics?

“Each of us has been doing statistics all his life, in the sense that each of us has been busily reaching conclusions based on empirical observations ever since birth.”

Kruskal