Active Learning in A Level Mathematics

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My aim is to give all of our students the opportunity to actively engage in their own learning. I use a range of classroom activities to encourage students to discuss, reflect, explain and think about their maths, to 'have a go' without worrying about getting a wrong answer and to explore the connections within mathematics.

Many of my activities involve matching or sorting cards whereby students typically work in pairs or small groups, and this can provoke a great deal of interesting mathematical discussion and thought. This is particularly so when the activity is open with more than one possible answer or can be tackled at different levels

One activity that is very popular with the students involves a jigsaw of equilateral triangular pieces; the side of one piece must be matched to another with an equal expression, eventually forming a large hexagon. I have also found that students enjoy and find it helpful to work on large sheets of paper with thick, coloured felt tips, and create posters. The posters are displayed in the classroom as a reminder of the solving of the problem. Students have also commented that the colours help them remember when it comes to revision.

I use a lot of questioning in my lessons where students answer using individual A4 whiteboards. This means the whole class is participating, and encourages students to answer a question they maybe aren't totally sure about as any embarrassment of answering in front of the whole class is removed and answers can be rubbed out afterwards. This can help to improve the confidence of many students, and allows me to observe the whole class at once and deal with any errors and misconceptions that have come up. In this format I particularly like to ask open questions, where there are many possible answers and students have to come up with their own individual answer. This explores the understanding that the student actually has, and allows them to come up with answers of different levels.

What is an activity?

An activity is something that:

- provokes students into discussing, explaining and thinking;
- challenges and interests students;
- gets students actively involved in their learning;
- results in learning;
- provides some immediate assessment;

An activity should:

- have a clear purpose / aim / learning objective;
- be challenging yet accessible;
- address common misconceptions / errors;
- replace something not just be an add on;
- encourage students to take 'have a go' and take risks;
- encourage students to justify their decisions;
- be appropriate.

An activity should not:

- be too easy;
- be overwhelming;
- be too hard;
- be tedious or repetitive;
- allow shortcuts.

An activity can be used to:

- introduce a new idea;
- move learning along;
- refresh ideas learnt earlier in the course;
- to connect ideas;
- consolidate.

It is useful if:

- there is a natural extension or generalisation;
- it can be tackled at different levels.

Sorting Activity

$y = x^2 + 2x + 4$	$y = x^2 - 5x + 4$
$y = 2x^2 - 5x - 3$	$y = x^2 - 4x + 4$
$y = x^2 + 7x - 3$	$y = 4 + 3x - x^2$
$y = x^2 + 5x - 2$	$y = 6x - x^2 - 9$
$y = x^2 - 3x - 1$	$y = x^2 + 10x + 9$
$y = x^2 + x + 3$	$y = x^2 + 4x + 4$
$y = x^2 - 2\sqrt{3}x + 3$	$y = 3x - x^2 + 7$

Circles for Grid

$(x-2)^2+(y-3)^2=4$
$(x-1)^2+(y-3)^2=4$
$(x-3)^2+(y+1)^2=4$
$(x+4)^2+(y+2)^2=4$
$(x-2)^2+(y-3)^2=1$
$(x+3)^2+(y-3)^2=18$
$(x+4)^2+(y+4)^2=1$
$(x-3)^2+(y-8)^2=20$

$(x+2)^2+(y-3)^2=9$
$(x-3)^2+(y+4)^2=9$
$(x-3)^2+(y+1)^2=9$
$(x+4)^2+(y+2)^2=9$
$(x-2)^2+(y-3)^2=16$
$(x+3)^2+(y-5)^2=16$
$(x+4)^2+(y+3)^2=25$
$(x+5)^2+(y-1)^2=16$
$(x-4)^2+(y-13)^2=2$

$(x+4)^2+(y+3)^2=16$
$(x-5)^2+(y+3)^2=16$
$(x-1)^2+(y-5)^2=9$
$(x-4)^2+(y-3)^2=9$
$(x+1)^2+(y-7)^2=16$
$(x+1)^2+(y+2)^2=16$
$(x+3)^2+(y-1)^2=20$
$(x-2)^2+(y-3)^2=12$
$(x+2)^2+(y-3)^2=10$

These circles have the same radius.	This circle intersects the y axis but not the x axis.	These circles pass through the origin.
This circle intersects both axes.	These circles have the same centre.	This circle touches the y axis.
These circles have centre (-2, 3).	This circle intersects the x axis but not the y axis.	These circles have radius 4.
The translation from one of these circles to the other is $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$	This circle does not intersect either axis.	This circle passes through the point (5, 12).

Multi-representation Activity

$y = (e^{x})^{2}$	$y = \dfrac{e^{4x}}{2e^{2x}}$
$y = \dfrac{2}{\sqrt{e^{x}}}$	$y = 2\sqrt{e^{x}}$
$y = \dfrac{e^{x}}{\sqrt{e^{x}}}$	$y = \dfrac{1}{e^{2x}}$
$y = \dfrac{2}{e^{x}}$	$y = \sqrt{e^{2x}}$

$y = e^{-2x}$	$y = e^{x}$
$y = e^{2x}$	$y = 2e^{-x}$
$y = 2e^{\frac{1}{2}x}$	$y = \frac{1}{2}e^{2x}$
$y = e^{\frac{1}{2}x}$	$y = 2e^{-\frac{1}{2}x}$

$\frac{dy}{dx} = -2e^{-2x}$	$\frac{dy}{dx} = e^{x}$
$\frac{dy}{dx} = 2e^{2x}$	$\frac{dy}{dx} = e^{\frac{1}{2}x}$
$\frac{dy}{dx} = e^{2x}$	$\frac{dy}{dx} = -2e^{-x}$
$\frac{dy}{dx} = -e^{-\frac{1}{2}x}$	$\frac{dy}{dx} = \frac{1}{2}e^{\frac{1}{2}x}$

$\int y dx = \frac{1}{2} e^{2x} + c$
$\int y dx = -4e^{-\frac{1}{2}x} + c$
$\int y dx = e^{x} + c$
$\int y dx = 4e^{\frac{1}{2}x} + c$
$\int y dx = -2e^{-x} + c$
$\int y dx = 2e^{\frac{1}{2}x} + c$
$\int y dx = \frac{1}{4} e^{2x} + c$
$\int y dx = -\frac{1}{2} e^{-2x} + c$

Examples of open questions suitable for answering in pairs on posters

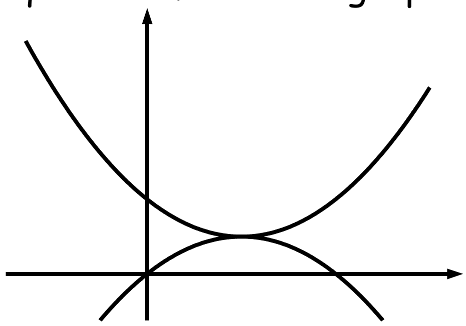
1. Justify each one as being the odd one out. Give as many reasons as you can each time.

$$y = 3x + 4$$
$$2y = 6x - 7$$
$$3y + x - 4 = 0$$

2. What is the same and what is different about the graphs of :

$$y = x^2 - 8x + 16$$
$$y = 4x^2 + 16x + 16$$

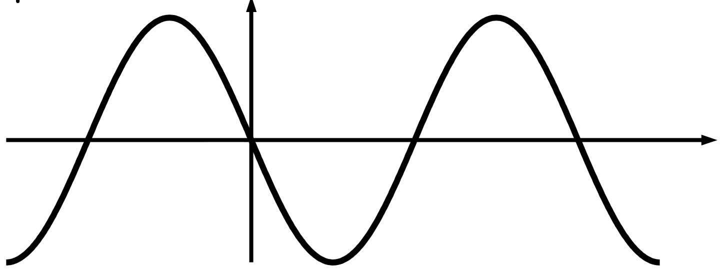
3. Find a possible pair of equations for these graphs and justify your choice:



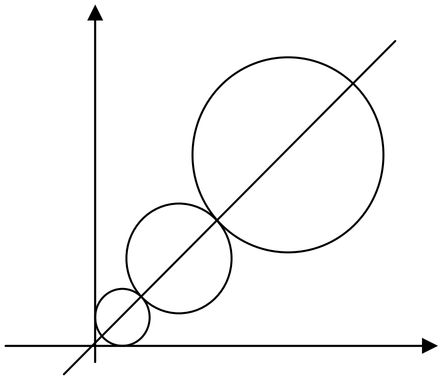
4. Use parts and substitution to find the integral below and show that both methods give the same answer.

$$\int x(x+2)^5\,dx$$

5. Give a many possible equations for this graph. For each possibility sketch the graph and mark the axes at appropriate points.

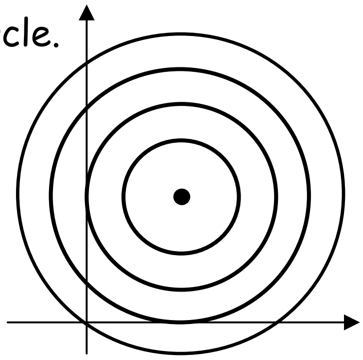


6. Give possible equations for these circles.



7. Give a possible equation of each circle.

Explain your answers.



8. Investigate this function and sketch its graph.
i.e. Find the x and y intercepts, stationary points and their nature and any other important features of the graph.

$$x^{\frac{1}{2}} + y^{\frac{1}{2}} = 1$$

or $$f(x) = e^{2x} - e^{x}$$

or $$y = \sin^2 x - \sin x$$

or $$f(x) = xe^{2x}$$

or $$f(x) = 2\sqrt{x} - x$$

Template 1 for the logarithm hexagonal jigsaw.

$\log_a 6^7$

$\log_a 2 - \log_a 7$

$\log_a 15$

$\log_a 0.375$

$\log_a x^2 y^5$

$\log_a 0.2$

$5\log_a 2$

$\log_a 36 + \log_a 2 - \log_a 4$

$\log_a 0.25$

$\log_a \frac{1}{8}$

$\frac{1}{3}\log_a 8$

$3\log_a 2$

$2\log_a 3$

$\log_a 30$

$\log_a 5 + \log_a 3$

$8\log_a 7$

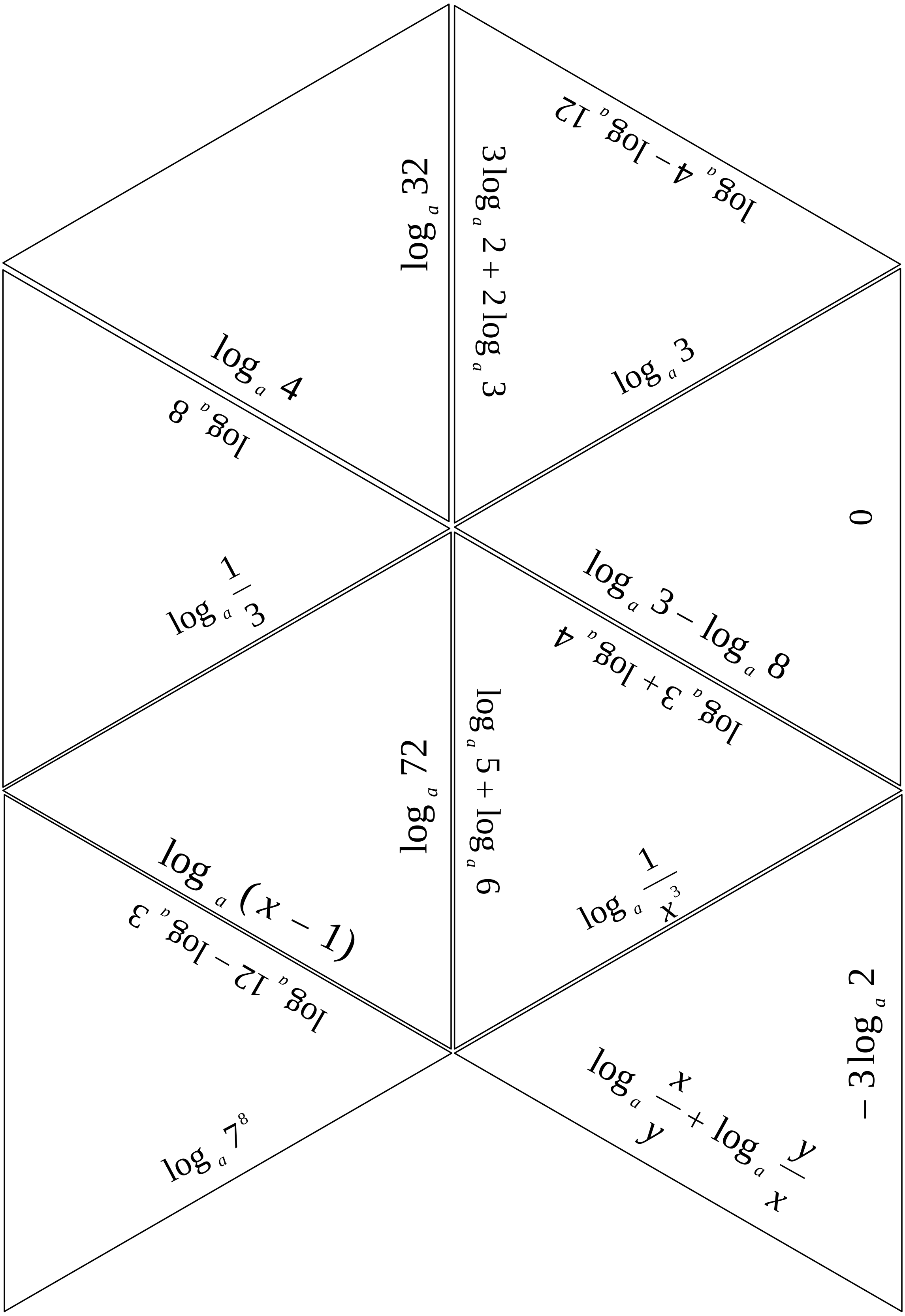
$\frac{1}{2}\log_a 7$

$6\log_a x$

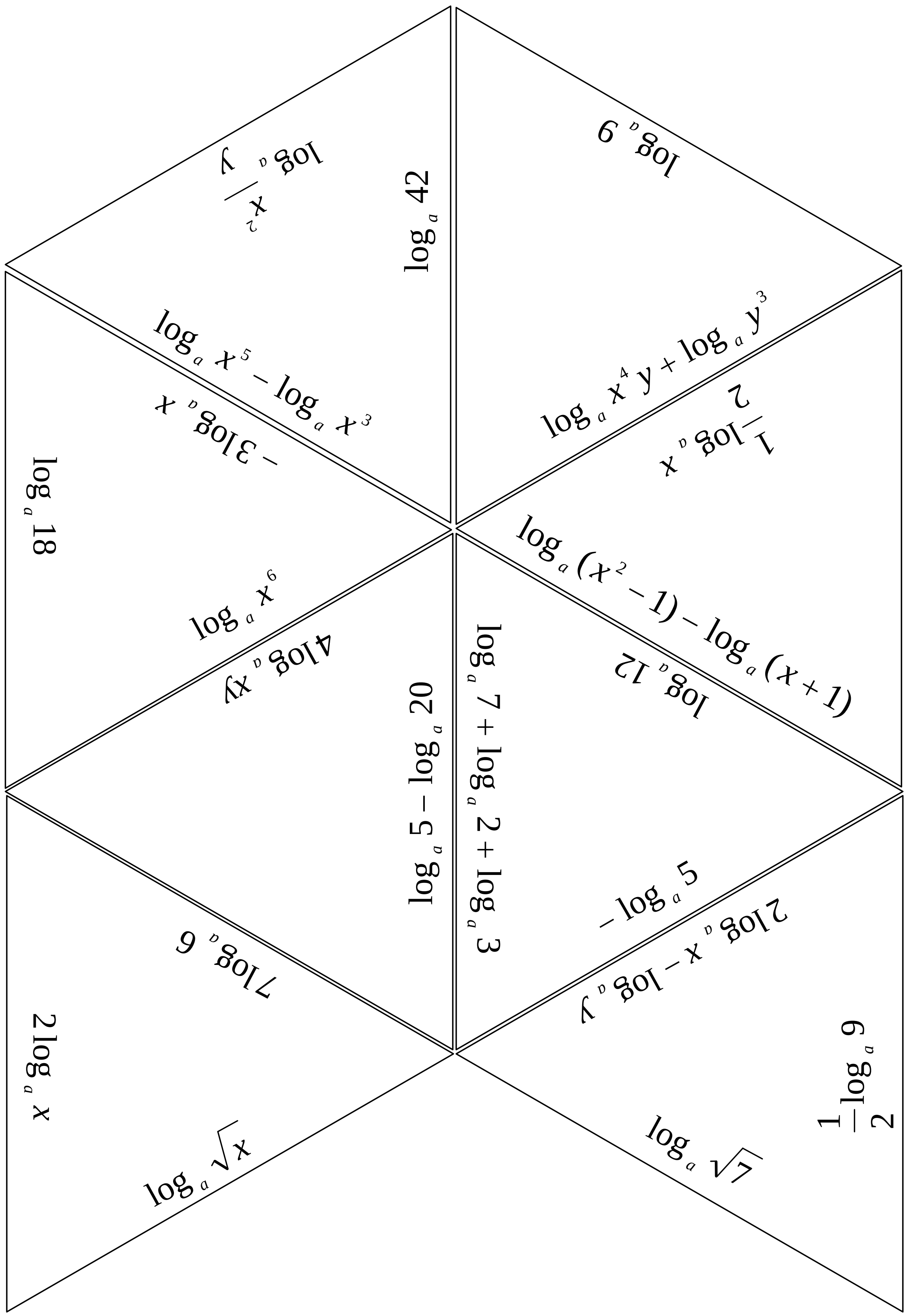
$2\log_a x + 5\log_a y$

$\log_a \frac{2}{7}$

$\log_a 2$



Template 2 for the logarithm hexagonal jigsaw.



Template 3 for the logarithm hexagonal jigsaw.