MEI Conference, July 2006

Some misconceptions and things often not done well in the MEI mechanics papers

It is sometimes difficult to tell whether a student has gone wrong because of a slip or because of a misconception. However, the following list of misconceptions is fairly widely held and when seeing them in a student's work one should be suspicious of there being more wrong than a simple slip.

Misconceptions

Vectors

Many students do not manipulate vectors with any confidence and do not have a clear concept of a vector equation.

Use of column vectors instead of the $a\mathbf{i} + b\mathbf{j}$ *form can help.*

Kinematics

Taking the direction of the motion of a particle from its position vector $\mathbf{r}$ instead of its velocity vector $\mathbf{v}$.

Not realizing that for a particle to be (instantaneously) at rest it is necessary for $\mathbf{v} = \mathbf{0}$.

Using constant acceleration formulae inappropriately and also using *distance* = *speed* × *time* when the acceleration is not zero.

Confusion of $x - y$ and $t - y$ graphs so that the direction of motion of a particle is described in terms of the shape of its $t - y$ graph.

Forces

Newton's third law is not well understood. Instances are

- drawing false normal reactions (e.g. at the point of contact of a string with a block),
- wrong arguments in lift problems.

Dynamics

Wrongly applying Newton's second law to connected particles and, in particular, not being able to find the force in a coupling.

Wrongly believing that if, say, a car and a caravan are decelerating then there must be a thrust in the coupling.

Projectiles

Wrongly believing that when a particle is fired at a fixed speed from an origin and there are two trajectories passing through a given point, that the particle

is rising on one trajectory and falling on the other,

is projected at an angle less than 45° in one case and more than 45° in the other.

Things often not done well (but not necessarily because of a misconception)

General

The use of clear diagrams for questions on statics, dynamics or kinematics (produced even if not instructed to draw them) so that positive directions (and an origin) are properly defined, thus reducing the occurrence of sign errors etc.

Recognising all the forces present and when their sense is determined by the situation (e.g. tension in strings, friction acting in a direction opposed to the motion).

Recognising that the situation being modelled has changed (e.g. removal of one force may change the value of another).

Correct resolution in *sensible* directions, especially the ones given in a question.

Using vectors appropriately.

Interpretation

Students are expected to be able to explain a result using a concise and sound mathematical argument with the correct use of mathematical language. They are also expected to understand the modelling process and be able to relate results obtained from the mathematical model to the situation being modelled. Students frequently reveal an almost total lack of understanding of the significance of their results. This is sometimes compounded by a lack of command of technical language so that terms like force, work, energy etc are often used as elegant variations in a piece of prose.

Students should be encouraged to use diagrams and equations to aid their explanations.

Algebra, trigonometry and calculus

The algebraic demands of the questions are not always negligible, especially in M2 and higher papers. Some questions are based on the exploration of a model where at least one of the quantities involved is a variable. Students whose simple algebra and trigonometry are not sound are likely to produce a confused mess very early in such questions and are not really ready for them.

Students should be able to deal efficiently and accurately with simultaneous equations in two variables and with quadratic equations.

Showing a stated result is true

Questions frequently ask the students to establish a given result in order to ensure that they have the correct expression for further use. The argument must be made **very** clear involving statements such as *resolving vertically*, *applying Newton's second law down the plane* etc. Methods tend to be especially sketchy when students are asked to formulate an equation of motion.

Statics

Diagrams are often not clear enough to allow subsequent resolution, false reaction forces are introduced and new diagrams are not produced when the situation being modelled is changed.

Many students seem to be able to resolve forces in a given direction but not be able to establish an equation given that there is equilibrium. Many students cannot draw a polygon whose sides represent the forces acting in a given situation; in particular, the triangle of forces is often not considered as a method of solution of a three force problem.

Dynamics

Generally, Newton's second law is applied accurately to single particles or bodies but connected particle problems reveal many misunderstandings. Many students are unable to separate the component parts when there are several particles present and then do not seem to be able to relate common sense to their analysis and are content to have one part of a system in equilibrium and a connected part accelerating.

Many students take friction to be opposing acceleration instead of motion. For example, suppose a sledge is being pulled along horizontal ground by a string and then the string breaks; the frictional force continues to oppose the forward motion even though the sledge is decelerating.

Kinematics

Whether or not vector notion is being used, the constant acceleration formulae are applied even when the acceleration is manifestly not constant. When vector forms are required many students struggle with them.

When formulae are quoted there is frequent confusion with the current value of 'u', 't' etc, so in a question which divides the motion into two parts the value taken for 'u' in the second part is likely to be that used in the first. The absence of a defined origin and of the direction being taken as positive often lead to confusion and sign errors.

Students need techniques such as the introduction of question specific variables to help them avoid confusion with the notation.

Projectiles

Many problems seem to stem from students learning formulae for horizontal range, maximum height or even the trajectory and then applying them inappropriately. The examination questions are usually structured to suggest an efficient method and no assumption is made that the student has memorised, say, the general trajectory formulae.

Many students, when using the '*uvast*' formulae, allow '*u*', '*t*', '*v*' etc to stand for different measures in the same question; this invariably leads to confusion.

As with general kinematics the students need techniques to help them avoid confusion with the notation. I would use x for the horizontal position, $\dot{x}$ *for the general horizontal component of velocity,* $\dot{x}_0$ *for the initial horizontal component of velocity and similarly for the vertical direction in terms of y. Some teachers use* u_x, v_x, s_y *to achieve the same distinction.*

Many students do not recognise situations where the result $v^2 = u^2 + 2as$ applied to the vertical motion gives an immediate answer to the question.

The absence of a diagram indicating the direction taken as positive frequently leads to sign errors, often followed by fudging. Many solutions have no indication of method, often making it impossible to tell which part of the motion is being considered.

Specific problems with the content of MEI Mechanics 2

Friction
Mistakes are often made with the direction of the friction force, with students not recognising that it is opposite to the motion not the acceleration. When situations change, causing a change in the normal reaction, this is frequently not followed through to give a new limiting value for the friction. Students often suppose that friction must always be limiting, even in conditions of static equilibrium; this error is inevitable when students write μR in place of the frictional force throughout a question.

This can be alleviated by never writing $F = \mu R$ *but instead writing* $F \le \mu R$ *and* $F_{\max} = \mu R$.

Many students believe that, in all circumstances $\mu = \tan\theta$ for a body sliding down a slope at θ to the horizontal.

Linear momentum
Most of the errors stem from the absence of diagrams clearly establishing a sign convention for the velocities or from an inappropriate application of work-energy equations. Students often neglect the opportunity to simplify their solutions when linear momentum is conserved throughout a multi-stage problem by using the information that the total linear momentum at each stage of the problem is the initial total linear momentum.

Moments
Questions involving moments very often lead to poor solutions, especially when the determination of the moment requires resolution. Many attempted solutions lack suitable diagrams, omit forces (especially at a pivot) and sometimes introduce phantom reactions where, say, a string is attached to a beam. A common mistake is to treat a heavy beam as if it were light and to assume that the internal forces act in the line of the beam. Omitted forces often lead students falsely to suppose that a problem can be solved by resolution alone. The conditions for toppling are often not well understood.

Pin-jointed light frameworks
As might be supposed, students who struggle with the application of techniques involving moments and the general principles of statics tend to founder given one of these problems. It is vital that you use a systematic approach if you are going to get these problems out efficiently (or at all) and a diagram is always required.

Centres of mass
When attempting to find the centre of mass of a structure, students who use vector methods are usually more successful than those who attempt an approach using moments, especially when three coordinates are involved. However, arguments based on moments are often neater and more direct for certain problems, for instance, ones involving the angle of a suspended structure. Many students seem to have a restricted range of techniques available. A common error is to use mass instead of weight when trying to calculate a moment.

A very common inefficiency is to continue to manipulate a composite figure as the sum of simple parts after the position of the centre of mass has been determined.

Work-energy
Many students struggle with the exact meaning of the language involved. The most common errors are to neglect the work done by one of the forces (usually the weight) or to use the component of a force inappropriately (e.g. use WD is $Fs\cos\theta$ and substitute a component for F or for s.

A particularly common error is to neglect part of a GPE change. Many students try to solve all of these problems using N2L; this method almost always involves more calculations, is more time-consuming and is frequently based on an incorrect analysis of the problem.

Specific problems with the content of MEI Mechanics 3

Simple Harmonic Motion
Not all candidates seem familiar with the language involved and there is frequent confusion between the use of degrees and radian measure. Many candidates have little idea about how to establish that a given SHM equation models given data and a large number are unhappy when a question requires them to investigate the mechanics of a system which is in SHM or to establish the equation of motion.

Elastic strings
The basic ideas are generally well understood. However, when calculating the work done in stretching a spring, many candidates do not consider the energy stored due to any initial stretching. Another common error is to overlook the total change in PE when a system moves in a vertical line; for instance, if a particle is attached to an elastic string and dropped from the point of attachment of the string, when calculating the distance fallen before the particle is first instantaneously at rest, many candidates will equate the energy in the stretched string to the PE lost only up to the point when the string first became taut.