Core 3 coursework using EXCEL

General tips for producing a spread sheet.

Remember:

- Set up a header/footer: use FILE, PAGE SETUP select HEADER/FOOTER and write in what you want to appear at the top or bottom of your print out.
- To align left/right or to centre: use the icons on the formatting tool bar (The pictures!!).
- To enter a formula: always use an equals sign first.

Addition: = SUM(A1 :A4)
or use
Multiplication: = A1 *A2
Division: = A1 / A2
Powers: = A1 ^ 2
(giving A1 squared)

- To copy a formula: highlight cell to be copied and the ones you want to contain the formula and drag the button at the bottom right of the cell down or FILLRIGHT (or use EDIT FILLDOWN.)
- To insert / delete a row/column: use EDIT INSERT/DELETE.
- To clear a particular cell use delete.

To widen a column: either use FORMAT COLUMN WIDTH or move your cursor to the heading of the column in question. When it takes the shape of a double headed arrow, click and drag to required width.

- To show your formulae use OPTIONS SHOW.

To help you set up a spread sheet for your own equations try the following.

Method 1:Change in sign.

Here we are looking for a change in the sign of y. If we let our first column be values of x our second must take that value and find y for us so:

If $y = x^5 - 5x + 3$ and we know one root is in the interval (l,2).

In the first column first cell enter 1, then in the cell beneath type = A1 + 0.1. Now fill this cell down the column until the number 2 is reached.

In the second column first cell enter the formula to give you y i.e. =Al^5 - (5*Al) + 3.

When you press enter you should get -1 as *the* entry in this cell.

- Now FILLDOWN this column

- The change in sign should be between 1.2 and 1.3 so we want to zoom in here.

In the A column, in the next available empty cell, enter 1.22 then 1.23 etc. in the others (you could just type = A5 + 0.01 and fill formula down).

In the B column we want to copy the formula for y as before. (highlight *B5* to B?).

Find the change in sign as before and zoom in to get an even more accurate value of the root (you should get *1.275...).*

Save the spread sheet using FILE SAVEAS.

Method 2: Newton-Raphson

For this we need to set things up carefully. We need:

$f(x)=x^5—5x+3$:– this will be Bl and also
$f(x)=5x^4—5$:–this will be Cl

- In A1 type in your first approximation 2.

In B1 type in *=A1^5 - 5*A1+3.* You should get *25.*

- In Cl type in *=5*A1^4 - 5.* you should get *75.*

- In D1 type in =A1-B1/C1. This is your first approximation from N-R and should be 1.666...

1.66.. is our next x value so copy it to A2 by putting =D1 in the A2 cell. Copy this formula by filling down (don't worry about the numbers you get yet!

Copy the other columns down in the same way. In the D6 cell should be the number 1.275682.

See if you can find the other roots of this equation using the spread sheets you have set up. Are all the roots found by each method?

Method 3: x = g(x).

Our root is approximately 1 so:

- First rearrange your equation so it reads x = (x^5+3)/5

- Using FILE NEW call up a new spreadsheet.

- In the first column first cell write in 1.

- In the next cell down we need to get our next approximation by writing in g(x). type = *(A1^5 + 3)/5.*

This should give you 0.8

- We want to let the next cells down to be *(A2^5 + 3)/5, (A3^5 + 3)/5* etc. Do this by highlighting and filling the formula down as we did for method 1. If you fill down as far as A11 you should get *0.618034....*

- Fill down further to get an even better approximation. Note that this rearrangement will only give the root between 0 and 1 and not that between 1 and 2 due to the gradient of g(x).

Autograph and Core 3 Coursework

The Basics

Autograph is an advanced graph drawing package which will make doing the C3 coursework much simpler than drawing by hand or with EXCEL. Start with familiarising yourself with the important menu options:

Adding an equation (the button with an = and + sign on it) Pressing this button lets you enter equations to plot, using the ^ key to register a power

Changing the Scale: clicking on **axes** then **edit axes** (or just press the spanner button!) lets you adjust the minimum and maximum x and y values.

Zooming: clicking on the magnifying glass buttons (+ or **-**) lets you zoom in or out repeatedly. The same results may also be achieved by pressing the + and – keys at the far right of the keyboard. The buttons alongside are perhaps more useful as they enable zooming in to an area you've first drawn a box around. The buttons to the right of these enable zooming in or out in the x and y directions only.

Undoing a zoom: there is an undo button. You may also return to the default axes you started off with (the button just to the left of the degrees button)

Selecting a curve: positioning the mouse alongside a curve until a black arrow appears and then clicking the mouse turns the curve black and selects it for analysis. To select a second curve, you must first hold down the SHIFT key on the keyboard, and then repeat the above.

Right-Clicking the mouse: gives several additional menu options, especially once one or more curves have been selected as above. The same options are displayed by clicking on OBJECT on the toolbar at the top once a curve is selected.

Slow-motion drawing: press the button which looks like a tortoise. All curves are now drawn in slow motion. The buttons alongside this one allow for pausing etc.

Copying into WORD: clicking on PAGE and then copy page option, enables you to subsequently paste the graph, its annotation and the relevant equations into WORD.

Display of Results: the results box is situated to the left of the spanner button. Pressing it will give a display of all results found during the session – quite useful for solving simultaneous equations and a host of other things!

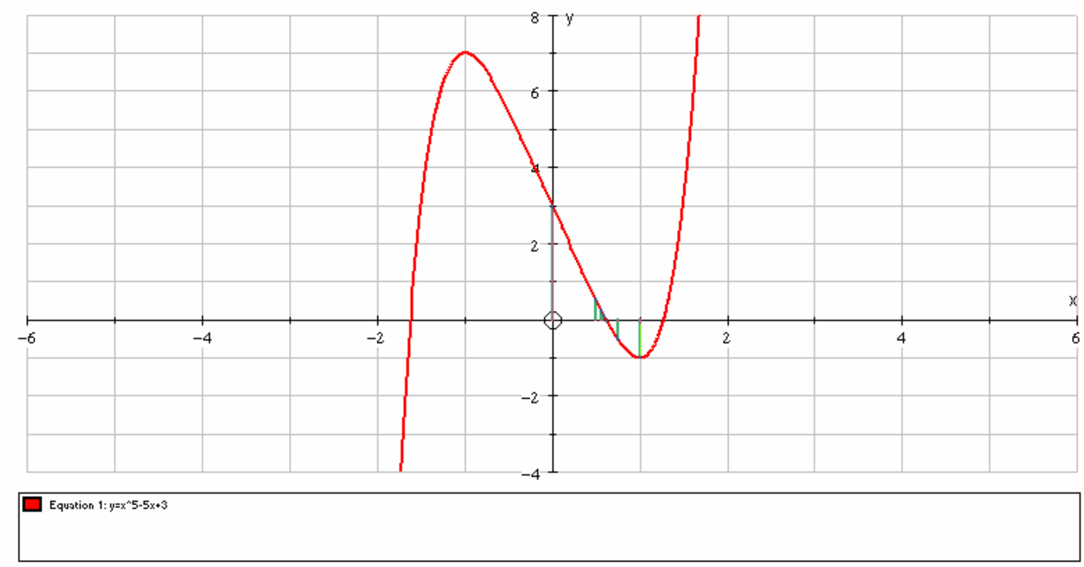
There are many other buttons, such as the equal aspect buttons, differentiation and integration buttons and many more, but this will do for now!

Use in Core 3 coursework:

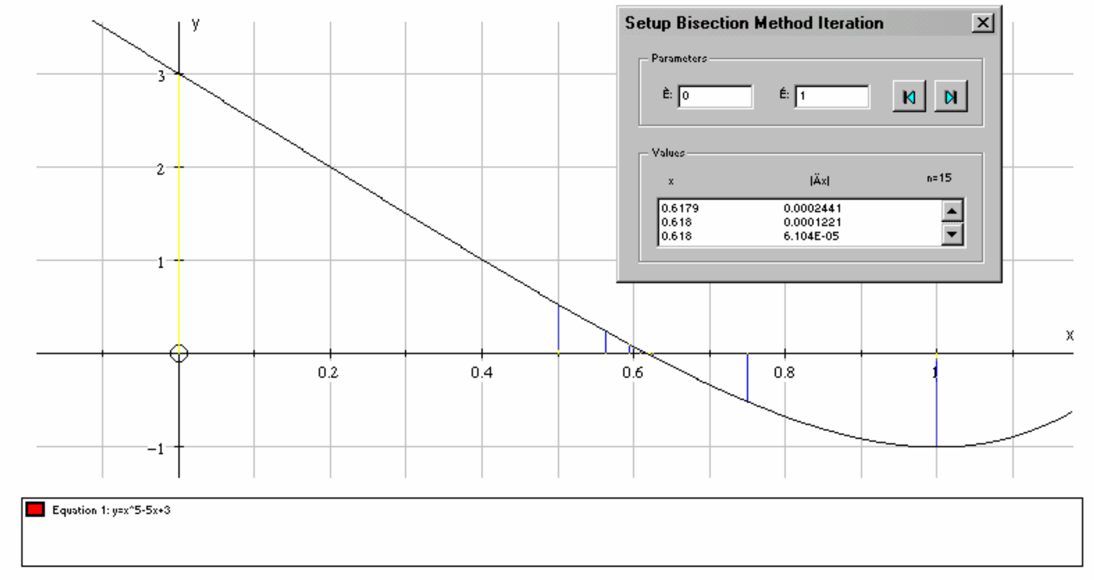
1. Change of sign methods

Autograph will draw the function whose roots you wish to find – just type in the equation in the Add Equation box and press OK, adjusting the axes to fit.
There is no doubt that decimal search is the easiest of the available techniques to use, and the zoom buttons on AUTOGRAPH are useful in illustrating the progress of the EXCEL spreadsheet in homing in on the root.

The picture below shows the change of sign in the function $y=x^5-5x+3$ between $x=0$ and $x=1$, illustrating that, as the function is continuous, there must be a root between these two x values.

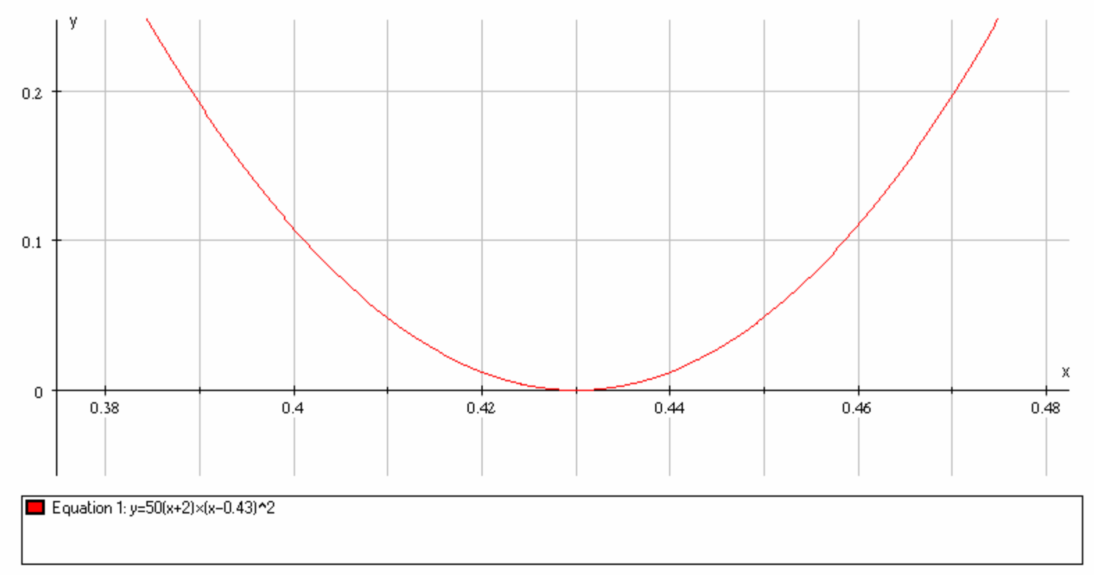


However, if you wish to use the Bisection Method, select the curve and then right-click the mouse. Choose Bisection Iteration and select the initial two x values. Clicking the forward arrow in the dialogue box shows the progress of the technique AND illustrates it on the graph at the same time.



Method Failure: Designing a function with repeated roots (done in the Polynomials chapter of C1) will give a function which touches the x-axis and thus the method will fail as no change of sign is detected. You should however, choose the repeated root to be a number to **at least 2 DECIMAL PLACES**. Otherwise the first table in the decimal search procedure will solve the equation on its own – giving a student no credit in this section.

e.g

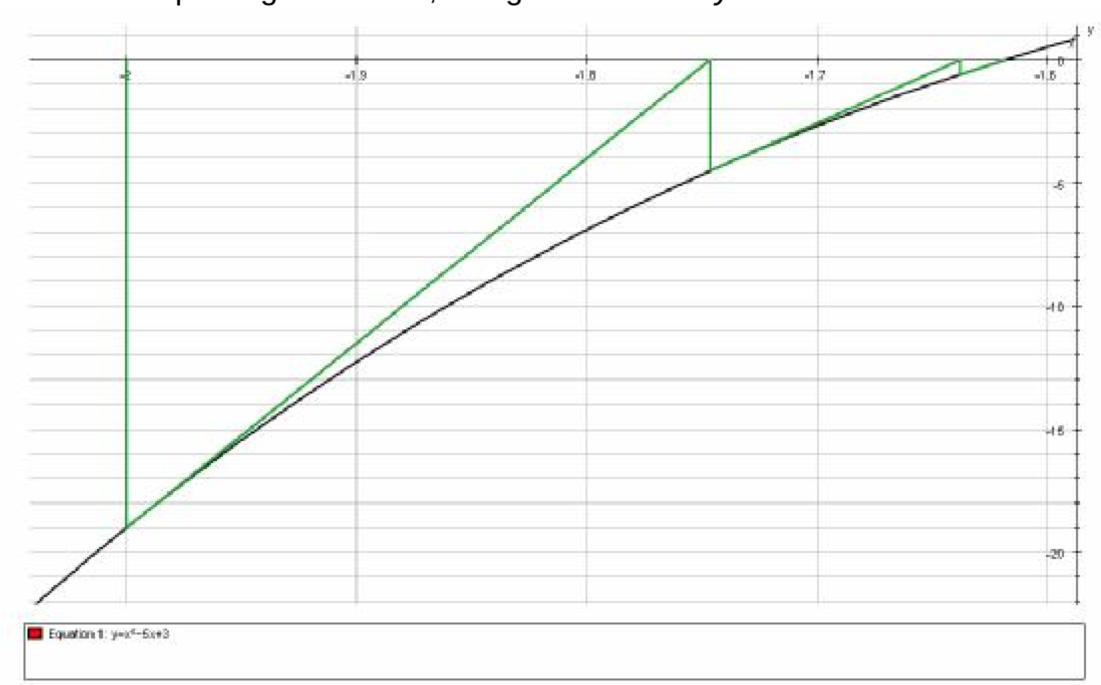


The 50 factor in the above example is simply to “sharpen” the vertex at 0.43 and emphasise the fact that the graph merely touches at this point.

Note: although the initial design of the failure is artificial and uses a squared factor, when writing up the coursework, give the equation in an expanded form (without brackets) – otherwise the solution is obvious by algebraic techniques.

2. Newton-Raphson: Having selected a curve and right-clicking on the mouse, now choose the Newton-Raphson option. Choose your starting point and click the forwards arrow repeatedly. The package draws the tangents and finds the root at the same time. This gives a good way of supplying the necessary illustration of the process required by the markscheme, without having to draw the tangents by hand.

An example is given below, using the function $y = x^5 - 5x + 3$



The results box will display the results below for the equation $y = x^5 - 5x + 3$,
$y = 0$, for the root between –2 and –1:

x	|D *x*|
-1.74667	0.253333
-1.63775	0.108912
-1.61858	0.0191753
-1.61803	0.000544541
-1.61803	4.29425E-007

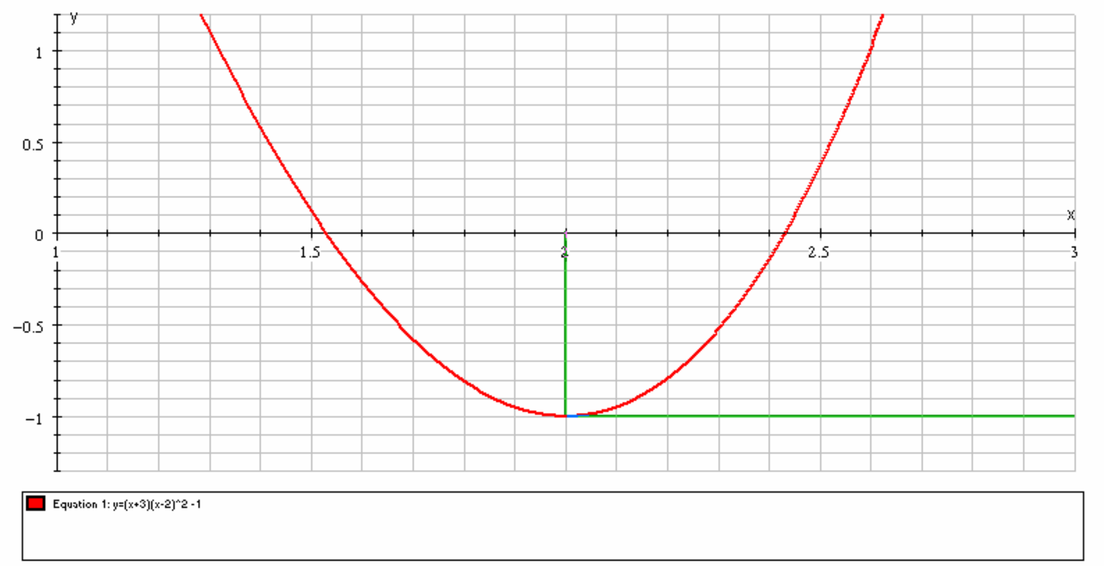
↑ root

↑ difference between one iteration and the next.

Newton-Raphson Failure: AUTOGRAPH is very useful in designing an equation to fail. The easiest way is probably to design say a cubic which has a repeated root at an INTEGER value. This has been covered in C1 in the Polynomials chapter. Now simply lift the curve up or down fractionally by adding or subtracting a small constant. You should now have one root, with a stationary point situated at an integer value just to one side – and thus a valid starting point under the Newton-Raphson technique. However, drawing a

tangent at such a starting point clearly gives a horizontal line which will not cross the x-axis and thus the method fails.
e.g. $y=(x+3)(x-2)^2 -1$

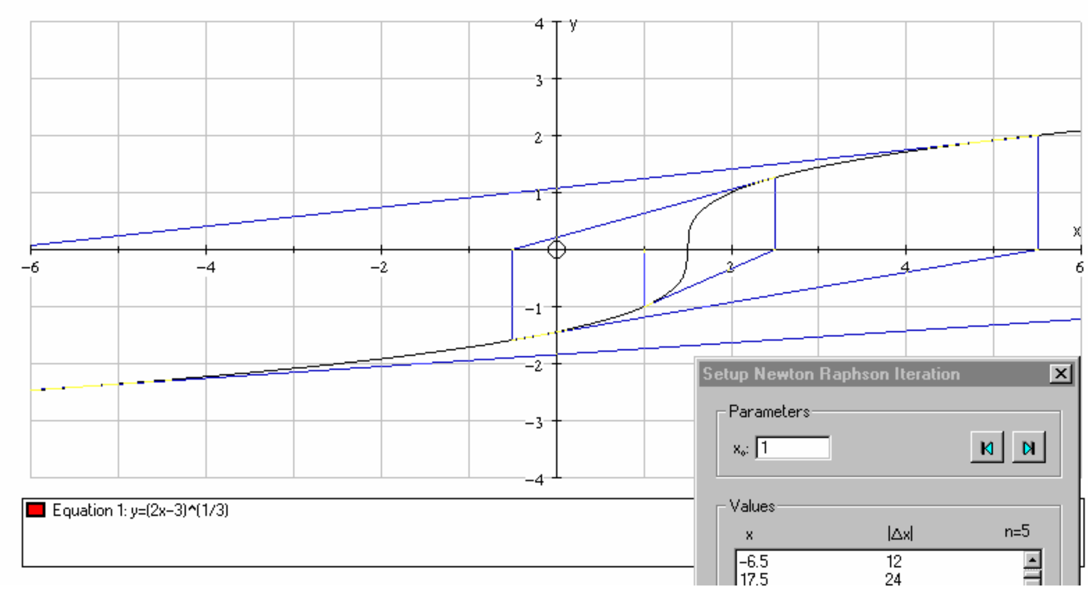
The method is illustrated below:



Note: to get AUTOGRAPH to draw a tangent, choose a starting point slightly to the right of 2, 2.0001 say: otherwise the results box displays a divergent result immediately and no tangent is drawn! This can also be rectified by increasing the number of significant figures the programme is working to.

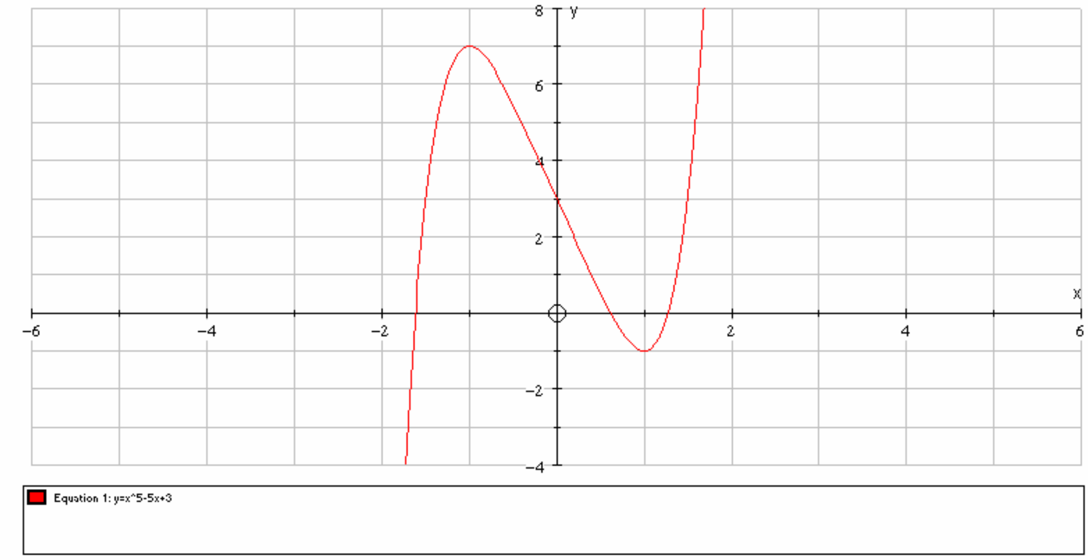
Alternative: for those with more advanced calculus skills, functions which are **odd roots of linear functions** are rather nice in their divergence:

E.g. $y=(2x-3)^{(1/3)}$



3. Rearrangement Method

An illustration of the roots of the original equation may first be given, using AUTOGRAPH e.g. for $x^5 - 5x + 3 = 0$

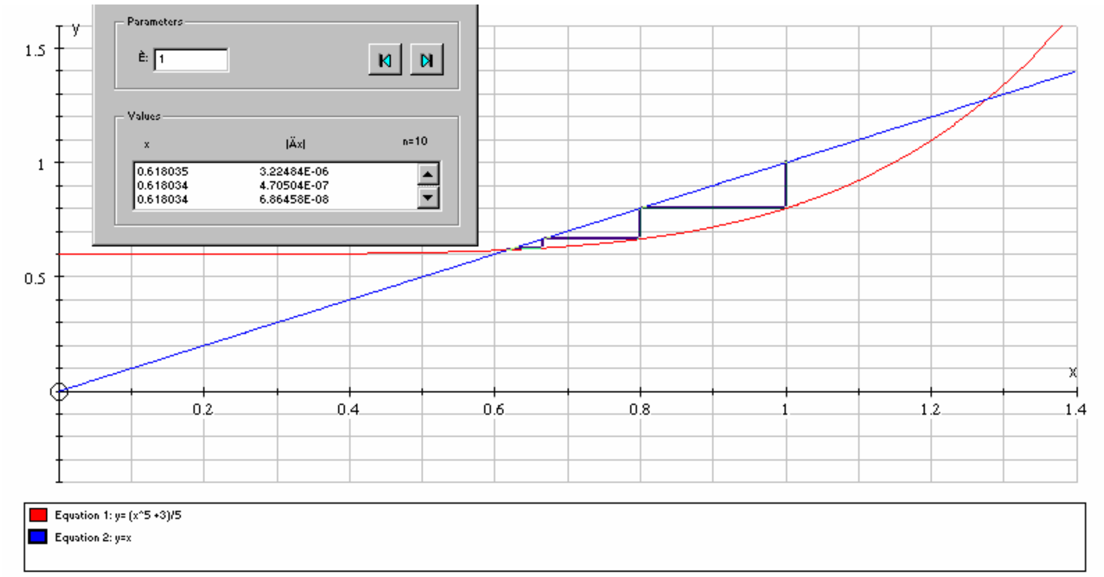


A rearrangement is then applied of the form $x = g(x)$. In the example thus we can see that one possible such rearrangement is:

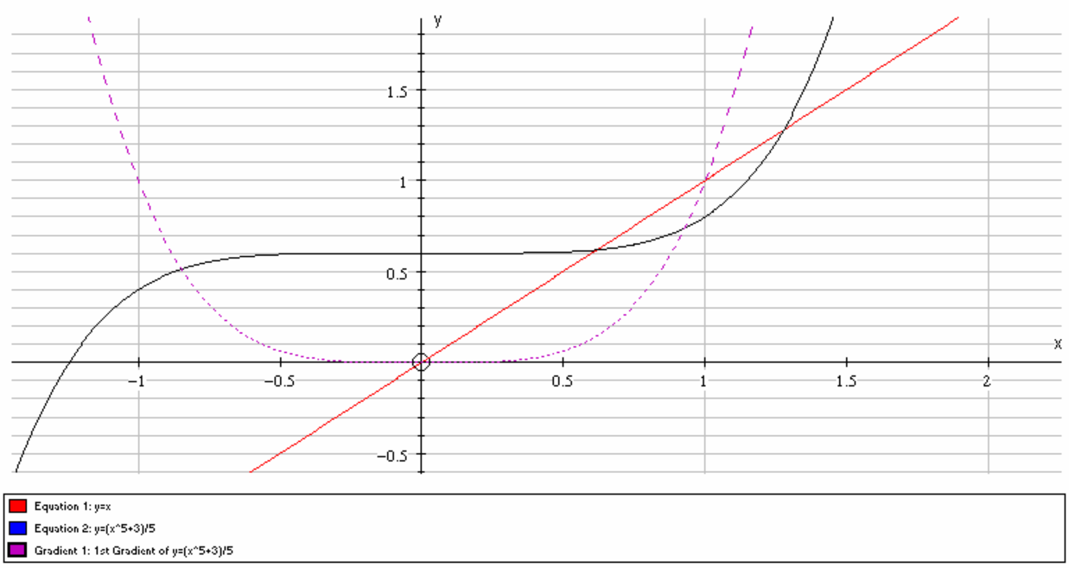
$$x = (x^5 + 3)/5 = g(x)$$

The method now turns this into an iterative procedure: $x_{n+1} = (x_n^{\ 5} + 3)/5$

Thus a value, x_0, is initially inputted into $g(x)$ to obtain the next value, x_1. The process is repeated until there is no difference between the iterations to the required degree of accuracy. On AUTOGRAPH, this is achieved by drawing both $y = x$ and $y = g(x)$ on the same axes, selecting them both, right-clicking the mouse and selecting the $x = g(x)$ option. Now choose a suitable starting point and press the forwards arrow:
This is illustrated in the graph below



For the process to succeed, $g'(x)$ must be less than 1 in magnitude in the vicinity of the root. This also may be checked on AUTOGRAPH by drawing the gradient function:



It can be quite clearly seen that the graph of $g'(x)$ (dotted) is less than 1 for $x \in (0,1)$ and thus the method succeeds here. Of course the failure of the method is illustrated on the SAME graph, by noting that the gradient of g(x) is greater than 1 in magnitude for the root between 1 and 2. Divergence can now be shown on AUTOGRAPH by changing your starting value: