

FP2 with only C1, C2 and F1 as a base

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Background

What we did before

Under the old specification we taught Further Maths successfully for many years. We seldom had sufficient Further Maths students to justify teaching them separately for all of their units, so generally taught their Single Maths A-Level together with those who opted for Pure and Mechanics, on the basis that this single option usually attracted the more able students. The overall level of ability meant that we were able to complete Pure 3 by January in the UVI. Their examination pattern was:

Examination Session	Units taught jointly	Units taught separately
January LVI	Pure 1	Statistics 1
June LVI	Pure 2, Mechanics 1	Statistics 2, D&D
January UVI	Pure 3, Mechanics 2	Pure 4
June UVI	Mechanics 3	Pure 5, Mechanics 4 or Statistics 3

We have never offered a mixture of ‘Applied’ disciplines to single maths students, so those taking Statistics had a slightly easier time, since the pace was not forced by having the Further Maths students in their group. Their examination pattern was:

Examination Session	Units
January LVI	Statistics 1
June LVI	Pure 1, Statistics 2
January UVI	Pure 2
June UVI	Pure 3, Statistics 3

Incidentally, this meant that when the last syllabus change was implemented, we were able to move all of these students to the current specification, only teaching the content for some of C3 and C4 in the UVI, carrying forward their results for S1 and S2, and sitting C1 and C2 in January UVI.

What we do now

The move to facilitate the completion of AS Further Maths in the Lower Sixth, by offering the new FP1, has had knock on effects that we, as a Maths Department, did not foresee. One of these is that there is, in my opinion, a bigger jump than ever to the compulsory Further Maths A-Level unit FP2. Our current programme is:

Examination session	Units taught jointly	Units taught separately
January LVI	C1	S1, Possibly D1
June LVI	C2, M1	FP1, D1 if not in January
January UVI	C3	S2
June UVI	C4, M2	FP2, DE or S3

Our timetable policy is that, where possible, two members of staff share the teaching of each A Level subject. Problems constructing the actual timetable meant that last year there were three, so instead of finishing S2 first, I was starting to teach FP2 at the beginning of the UVI, since that was the only unit for which I was responsible. It was only when I started to think about the most user-friendly order of topics that the paucity of the students' Pure Mathematics really hit me. I am not an advocate of the 'Here is a result. Use it and you will find out more about it later' method, although at times I was sorely tempted!

One big problem is that the foundations for most of FP2 mathematics are in C4, not even in C3 – by page 3 students need a facility with the trigonometric identities from C4 that can only be obtained from practice.

Similar subject matter was needed for Chapter 2 on Polar co-ordinates, once the initial work on sketching the curves was done, and for Complex numbers in Chapter 3! Had trigonometry been a C3 topic it would have been easy to ask colleagues to vary the order so that the FP2 course could proceed more smoothly. But it was not, and as C3 was the coursework unit I did not feel able to ask for a future topic to be brought forward.

Another problem, that I had not thought about in advance, is that although FP1 covers many similar topics to the old Pure 4 the level is nowhere near as challenging so the students' general mathematical skills are not as well developed.

The chapter on Matrices was my saviour. Here was material that only needed the content of FP1, the Chapter was long with 6 exercises associated with it and breathing space was granted while the students gained the necessary mathematical maturity to tackle the rest.

What I taught, and when.

Chapter 5 – Matrices

- Starts with work on determinants. Once the method had been introduced I tended to concentrate the manual method on algebraic determinants and use a calculator for questions that were purely numerical. Ex 5A and 5B were fine, although Ex 5B Q9 caused some discussion.
- Inverse of a matrix is again quite mechanical. Ex 5C, Q11, although starred (generally an indication of harder questions that may go beyond the usual examination standard) is very accessible.
- Connecting matrices with simultaneous equations follows on nicely from FP1, the triangular prism case of inconsistency being illustrated best with a ‘Toblerone’ bar! The concept of solving equations in terms of a parameter needed a bit more explanation since students had not done C4.
- I love the way the textbook introduces eigenvalues and eigenvectors, particularly Figure 5.16 on page 105. It takes what otherwise can be a very mechanical operation and injects some life into it. Ex 5E Q1-6 are fine for students to tackle, once they have understood the topic, Q7-10 were found to be a bit daunting and were discussed with staff help, and Q11-13 were left at this stage.
- Again, reduction to diagonal form is a routine topic and Ex 5F Q1-12 look, and are, fine. Q13 is not difficult, but looks daunting to all but the most able. In my opinion the starred questions are best omitted at this stage.

Chapter 4 – Power series

- This is introduced with a very palatable section on polynomial approximations using $y = e^x$ as an example. Although Ex 4A appears straightforward, my group did find problems where I had thought none existed! After finding some routine examples in other textbooks to restore confidence, the rest of this section was put on hold and we moved on.

By this stage we were nearing the end of the Autumn Term (I only had one 1h 10min lesson per week that term). All the group were preparing for Oxbridge interviews, (but only one for maths) and that, together with the actual interview times meant that lessons and homework were rather sporadic. I elected to move to Chapter 2 next.

From January I had two lessons per week.

Chapter 2 – Polar co-ordinates

- The concept of polar co-ordinates is not difficult and therefore an easy one to manage if you know that your class is going to be incomplete. After initially drawing graphs on polar graph paper, we quickly moved to graphic calculators to get the general shape of the curve. Drawing those parts of the curve for which r is negative using a dotted line does require more thought and the explanations with Example 2.2 are particularly helpful.

- Since Ex 2C is only a short one and completes the chapter, I did deviate from my plan to use only results that they had already met in their Single Maths lessons and introduced $\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$ and similar results, in order that we could complete the section on polar areas. Since the group had still not encountered parametric equations, we left Q8-11 until later. These questions are not strictly within the specification limits and I feel they should be starred.

Chapter 3 – Complex numbers

- This follows nicely from Chapter 2, providing an argument that the chapters might be better in the published order if the required initial knowledge is in place! Modulus and argument have already been introduced in FP1 and the work on loci follows on nicely. The published study plan only suggests a few easy questions from Ex 3A, but there is much to be learnt from the later questions in this exercise.
- By this point the students had done C4 Ch 8, so were familiar with the trigonometry necessary for multiplication in the Argand diagram, and de Moivre's theorem.
- Although the students were reasonably adept at manipulating complex expressions by this stage, comments in the textbook such as 'This means that the exponential function is periodic with the imaginary period $2\pi j$ ' worried some of them, and the beauty of $e^{j\pi} + 1 = 0$ was lost on all but the best! It is worth spending a little time at this stage showing how it is all consistent with established ideas. For example, using $e^{j\theta} = \cos \theta + j \sin \theta$, reduces de Moivre's theorem to $(e^{j\theta})^n = e^{jn\theta}$ - a result they could have found palatable years ago!
- The method suggested in Example 3.6 is worth emphasising – it provides an elegant solution that students are quite likely to miss.
- The work on summations in Ex 3G is a rich source of examination questions.
- Roots of unity can be dealt with fairly quickly, but the extension to the general case, and the associated geometry, proved more testing. We worked through the later questions in Ex 3I in class, as we did for 3J.

Chapter 1 – Calculus

- We didn't start this until early March, by which time C4 Ch 10 on further integration had been completed. At this point Ex1A proved relatively straightforward.
- Ex 1B Q10 is not directly in the syllabus, but is worth doing if there is time.
- Make sure that your students are aware of the contents of the examination formulae book. The tables of derivatives and integrals on p4 are invaluable. With those, any substitutions required in Ex 1C and 1D become much more obvious and the questions verge on routine.

I next gave my group the choice of 'Hyperbolic functions' or 'Investigation of curves', but with the latter taking up 74 pages in the textbook as opposed to only 14 pages for hyperbolic functions there was no contest!

Chapter 5 – Hyperbolic functions

- By now we were ‘on a roll’! Again, the tables on p4 of the formulae book are invaluable, and this topic did follow on nicely from Chapter 1.
- Osborn’s rule is useful, but only if students are secure with the trigonometric formulae in C4 Chapter 8
- The reference to Maclaurin series in Ex 6B Q9, 10, 11 provided a reminder that Chapter 4 had been abandoned earlier.
- Much of Ex 6C was very similar in appearance to Ex 1D which helped to allay fears that the ‘option’ might be something that was harder than the rest.

Chapter 4 – Power series again

- Ex 4B and 4C were much less daunting at this point, although they were done in class since it was the end of April by this stage and homework time was being used for revision papers.

What I might change next time

Next year I will be teaching S2, DE and FP2 so there will be much more flexibility. In addition I may do S3 and FP3, depending on how much progress is made. I started S2 in the post-exam time and propose to finish that first. I will then do DE and FP2 concurrently.

I don’t see any alternative to starting FP2 with Matrices, and the only major change that I envisage is to move Chapter 4 until after Chapter 1.

We will not have this problem the following year, since the Further Maths group is large enough to be taught on their own for their Single Maths lessons as well.

MEI online resources – FP2 provision

Subscription to this resource starts at £250pa, including MEI membership, access to the teachers' resources, the student record system, 2 staff passwords, and 20 student passwords. For centres needing more passwords the price increases on a sliding scale. For example, 6 staff and 100 student passwords costs £430.

If your centre does not already subscribe to this, it is well worth the investment.

Using the resource is very easy. It is laid out to follow the chapters of the book, the subject matter for each chapter being subdivided into up to five sections, depending on the length of the chapter.

Each section contains:

- A 'Study plan' giving information about what to read interspersed with suggestions of examples to try.
- 'Notes and examples' to supplement the textbook.
- A list of 'Crucial points'
- A 'Multiple Choice Test'. Students submit answers to these online. Each of ten questions has five possible responses, including 'I don't know'. The test score, based on 3 marks for a correct answer, 1 for 'I don't know' and 0 for an incorrect one is automatically written to the student's record. To pass requires a score of at least 23. A student can download fully worked solutions only after passing the test.

Most sections contain hints (at three different levels) for some of the textbook questions and some fully worked solutions.

Associated with each chapter there is a 'Glossary' and a 'Chapter Assessment' which is a written paper of examination-style questions.

The resource was developed to help Further Maths students who have little of no staff support in school, but it is just as useful for those who do.