MEI

Mathematics in Education and Industry

MEI STRUCTURED MATHEMATICS

C3 COURSEWORK

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What’s to be done and how?

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Pure Mathematics C3 coursework.

What is Pure Mathematics C3 coursework designed to do?

- Coursework assesses material which is unsuitable for a short modular examination. That is, that they should be able to solve equations efficiently, to any required degree of accuracy, using numerical methods.

- The intention is to give students a feel for the topic.

- It is designed to enable students to investigate methods and equations which is rather more than simply finding the answer to a problem. It is designed to enable students to appreciate how to use appropriate technology and to have an awareness of their limitations.

- Students should be able to use terms such as equation, function, root and solution appropriately in their write-up.

- Students should show geometrical awareness of the processes involved.

Requirements

Candidates must solve equations by the following three methods:

1. Systematic search for a change of sign using one of the methods

   bisection,

   decimal search,

   linear interpolation.

   One root is to be found together with the error bounds.

2. Fixed point iteration using the Newton-Raphson method. The equation selected must have at least two roots and all roots are to be found. Error bounds are to be established for one of them.

3. Fixed point iteration by rearrangement of $f(x) = 0$ into the form $x = g(x)$.
   One root is to be found.

A different equation must be used for each method.

In addition, a candidate's write-up must meet the following requirements:

(i) One of the roots must be found by all three methods.

(ii) The methods must then be compared in terms of their efficiency and ease of use.

(iii) There should be a graphical illustration of how the methods work on the candidate's equations.

(iv) Error bounds must be given for the root found by the change of sign method to within 0.5×10^{-3} and for one of the roots found by the Newton-Raphson method to 5 significant figures.

(v) For each method an example should be given of an equation where the method fails; that is, an expected root is not obtained or a root is not found. There must be an explanation, illustrated graphically, of why this happens. In this situation it is acceptable to use equations with known analytical solutions providing they are not trivial.

The mark scheme

- It is very prescriptive – to help both student and assessor.
- Half marks may be given throughout, but the total must be rounded (up or down) so that an integer mark is submitted.
- Graphs may be computer drawn, copied from a graphical calculator display, sketched or drawn from plotted points.

It is designed to be used at any stage of the C3 course.

It is not necessary for students to complete their work within a specified time. In other words the first section can be completed before the work on Newton-Raphson has begun. This section can then be completed at a later stage.

General points

Equations

Students may either make up their own equations or choose from a list provided by their teacher (or do a mixture). Any list provided should be sent with the scripts to the external moderator.

A different equation is required for each of the three methods.

Equations for demonstrating success should not be evidently analytically solvable (eg no quadratics). However, complicated curves are not necessary. A seventh order polynomial equation will not serve the student any more effectively than a cubic.

Students should show an appreciation of the difference between expressions, functions and equations. Repeated failure in this respect should incur a penalty of the loss of the terminology mark; some, not necessarily consistent, errors should be penalised by ½ mark.

Graphical work

It is essential that students demonstrate an understanding of what is going on graphically. (Hence the often seen confusion between function and equation, since a graphical demonstration of what is going on with an equation $f(x) = 0$ requires the graph of the function $y = f(x)$.)

Graphical illustration of the method means an annotated sketch with accompanying explanation, for the equation under consideration. A mere sketch is inadequate, and general bookwork is inappropriate.

Error bounds

Error bounds need to be established, not just stated, by showing a change of sign of $f(x)$.

Arithmetic demonstration

Examples of both success and failure of methods should always include some values of iterates showing the convergence or divergence.

A sensible level of accuracy is required. 1 decimal place is insufficient, 8 decimal places is too much. The syllabus mentions 0.0005 error for change of sign and 5 significant figures for Newton-Raphson.

Failure of methods

Equations for demonstrating failure should not be trivial, and a method has not failed if an initial table of values shows the roots. In general equations with no roots should not be used as examples of failure, but following the syllabus p51 an equation (of sufficient complexity) for which the method finds a false root is acceptable.

Oral communication

For all MEI Coursework there are three options; presentation, interview and discussion. A presentation is not appropriate in this module. The most appropriate would be an interview. Teachers should not only discuss the work before them but also challenge the student to work one of the methods in order to see if he or she understands the method. Failure to demonstrate an understanding that might be implied by the work within the report should be treated as suspicious.

Assessment

The overall mark awarded should reflect the quality of understanding and general worth of the script, as well as precisely how well the criteria have been addressed. Sometimes this will lead to leniency being allowed with respect to less important flaws.

Not every calculation needs to be worked, but within every script something needs to be checked, if only a confirmation that the root found does satisfy the equation.

Use of hardware and software

The use of computer and calculator resources is to be encouraged. However, candidates must take care that the graphs and calculations produced are relevant. Some will work the whole investigation on an ordinary scientific calculator, others with a graphical calculator, others in Excel or some other computer package.
Students need to be discouraged from spending a long time on writing up their work on a word-processor and especially if they are unable to use an equation editor.

An equation written 2x^3-x^2-1=0 is not easily read, while $2x^3 - x^2 - 1 = 0$ is.

Students might be encouraged to learn how to use Mathtype!

Points related to the specific domains

Change of Sign

Only one of the possible methods needs to be demonstrated.

Ideally the sketch should be annotated to illustrate the method. However, a good explanation accompanying a sketch of their graph is adequate, even without annotation. “Zooming in” on the range of the root is satisfactory.

Error bounds are inbuilt into the procedure, but the value of the root must be given with reference to these error bounds.

Failure may be illustrated using a false root for a non-trivial equation, or roots very close together, or multiple roots at a non-integer point. Typical equations deemed to be trivial are:

$$\frac{1}{x} = 0, \quad \frac{1}{(x-3)} = 0, \quad (x-2)^2 = 0$$

Newton-Raphson

All the roots are required so the implication is that the equation must have at least 2 roots.

The illustration of the method should show successive tangents converging on the root, for their specific graph. The formula does not need to be derived.

Error bounds must be established. It does not follow that because two successive iterates agree to 5 decimal places, that the value of the root is correct to those 5 decimal places. They should be established by change of sign.

Suggestions for the type of equation to demonstrate failure may be supplied. The starting point for failure should be the nearest integer on either side of the root.

Solving $x = \mathrm{g}(x)$

Only 1 root needs to be found.

The failure may be done in one of two ways:

(i) Demonstration that the chosen iterative formula used to find successfully one root will fail to find another.

(ii) Demonstration that a different iterative formula will not find the root already found.

If an equation is used in this domain with more than one root, the students may well find (i) the easiest option, but if the equation only has one root, then (ii) is the only option!

The explanation of failure need to be more than a mere cobweb/staircase diagram. I.e. there should be some attempt at least briefly to explain the convergence/divergence.

There is no need to calculate a value for $g^1(x)$, though this is an acceptable approach. However, a statement of the criterion for convergence is inadequate – reference needs to be made to the particular sketch. If the gradient is negative more care is needed to establish whether or not the criterion is satisfied.

Written communication

Students who persistently use the wrong terminology should not be awarded this mark. If there is some correct terminology then ½ mark may be awarded.

Comparison of Methods

One of the equations should be selected, and sensible comparisons will use the same starting point and the same level of accuracy.

It is reasonable to expect some quantitative statements such as NR takes 3 iterations, change of sign takes 4 etc.

It is reasonable for students to base their judgements on the equations they happen to have used. However, good answers will refer at least briefly to general considerations such as potential difficulties in finding a rearrangement that works.

Students are also expected to offer a brief discussion as to the ease of the use of whatever software and hardware they have used. For instance on a non-programmable calculator 10 iterates of an iterative formula might be very much easier than 5 iterates of the Newton-Raphson formula.