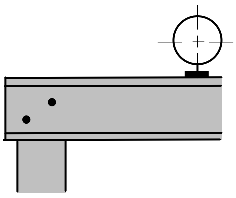
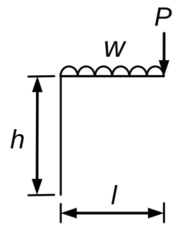
Use of Pythagoras's Theorem and Trigonometry in Analysis of a Bolted Connection

I am working on a project within the Design Applications department of Wood Group PSN during my Year in Industry placement. WGPSN are a leading service provider in the oil, gas and energy industry. My project involves the development of a function and user-input form for CAD software utilised by the company to aid the design of pipe supports. One aspect of the supports requiring analysis is the integrity of bolted connections. This can be done through several methods although one of the simplest is comparing the combined total force to which the bolts are subjected and their capacity.



This example looks at checking two 16mm diameter bolts for an 'L'-frame

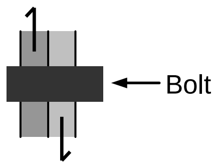


P	: point load (PL) - weight of the pipe	= 16.5 kN
w	: uniformly distributed load (UDL) - unit weight of beam	= 0.23 kN/m
l	: length of beam	= 510 mm
h	: height of beam	= 447.3 mm

There are two different forces generated by both the point load and uniformly distributed load:

- Reaction: Direct downwards force created by weights of the pipe and beam
- Bending Moment: Torque – force (weight) x distance (from bolts). For UDLs the distance is taken from the mid-point of the load to the bolts.

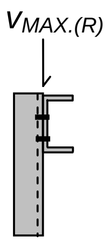
Both create a shear force on the bolt. This caused by the two beams moving independently; subjecting the bolts to opposing forces. This is the principle scissors utilise to cut through material.



To calculate the total combined shear force on the bolts you need to summate the horizontal then vertical components of the forces independently and combine.

Forces Generated:

Reactions:



$$v_{PL} = P = 16.5\,\text{kN}$$

$$v_{UDL} = wl = 0.23 \times 0.51 = 0.12\,\text{kN}$$

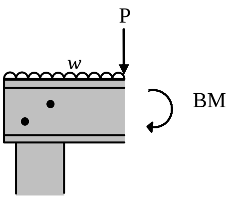
$$v_{MAX.(R)} = v_{PL} + v_{UDL} = 16.5 + 0.12 = 16.62\,\text{kN}$$

$$\text{shear per bolt} = \frac{v_{MAX.(R)}}{no.\,of\ bolts}$$

$$F_{v(R\,vert.)} = \frac{16.62}{2} = \mathbf{8.31\,kN}$$

The reaction force has no horizontal component. $\therefore\ F_{v(R\,hori.)} = \mathbf{0\,kN}$

Moments:

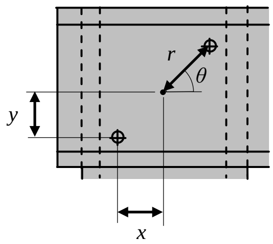


$$m_{PL} = PL$$
$$= 16.5 \times 0.51$$
$$= 8.42 \text{ kN.m}$$

$$m_{UDL} = \frac{wl^2}{2}$$
$$= \frac{0.23 \times 0.51^2}{2}$$
$$= 0.03 \text{ kN.m}$$

$$BM_{MAX} = m_{PL} + m_{UDL}$$
$$= 8.42 + 0.03$$
$$= 8.45 \text{ kN.m}$$

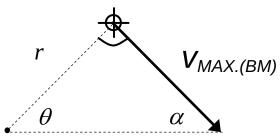
V_{max} is the maximum shear force generated by BM and occurs in outermost (both) bolts:



$$\theta = 45°$$
$$r = 35$$
$$x = 35\cos 45$$
$$y = 35\sin 45$$

$$v_{MAX.(BM)} = \frac{M.r}{\Sigma x^2 + \Sigma y^2}$$
$$= \frac{BM_{MAX}.r}{2 \times r^2}$$
$$= \frac{M}{2 \times r}$$
$$= \frac{8.45 \times 10^3}{2 \times 35}$$
$$= 120.7 \text{ kN per bolt}$$

Split into vertical and horizontal components using trigonometry:



$\theta = \alpha$

$$F_{v(BM\,vert.)} = v_{MAX.(BM)} \sin\alpha$$
$$= 120.7 \sin 45$$
$$= \mathbf{85.3kN}$$

$$F_{v(BM\,hori.)} = v_{MAX.(BM)} \cos\alpha$$
$$= 120.7 \cos 45$$
$$= \mathbf{85.3kN}$$

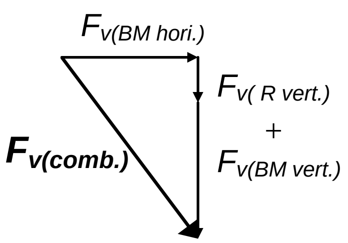
Combined:

Summate the horizontal then vertical components of the reaction and bending moments.

$$F_{v(h)} = F_{v(BM\ hori.)} + F_{v(R\ hori.)}$$
$$= 85.3 + 0$$
$$= 85.3 \text{ kN}$$

$$F_{v(v)} = F_{v(BM\ vert.)} + F_{v(R\ vert.)}$$
$$= 85.3 + 8.3$$
$$= 93.6 \text{ kN}$$

Use Pythagoras's theorem to determine the total resultant force:



$$F_{v(comb.)} = \sqrt{\left(F_{v(BM\ hori.)}\right)^2 + \left(F_{v(R\ hori.)}\right)^2}$$
$$= \sqrt{85.3^2 + 93.6^2}$$
$$= \mathbf{126.6kN}$$

The bolts used can withstand 29.4 kN in shear. **126.6 >> 29.4 kN ∴ bolts would fail**
Larger bolts or different bolt configuration should be chosen.