

Rotational Leg to Hull Stiffness for Tubular Legs in a Jack Up Rig

I work for GL Noble Denton, which is a consultant to the oil and gas industry. I undertake analysis of Jack Up offshore oil drilling rigs (see Figure 1) at different locations around the world using an in-house software package, which models the rig as a series of complex springs. The program simulates a storm and then calculates the dynamics and utilisations of different parts of the rig.

The rigs usually have three legs, which use a rack and pinion system to jack the rig up and down. The legs are fed through the hull by metal plates called guides. Some rigs have tubular legs, which are essentially large, long hollow metal pipes.

The problem is how to model the stiffness of the leg to hull connection (see Figure 2) when there is a rotating moment on the leg caused by environmental loading such as the wind, current and waves in a storm. Usually, the analyst would have to create a Finite Element model of the leg and jacking frame to find the stiffness, but as a first pass approximation, it is possible to use mathematical formulae and relationships to get a ball park figure, which is suitable in most cases.

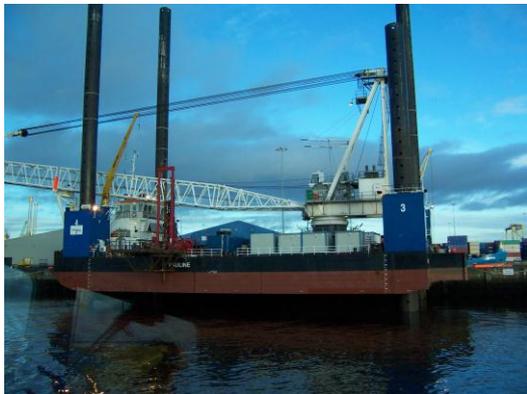


Figure 1 - Tubular Leg Jack Up Rig

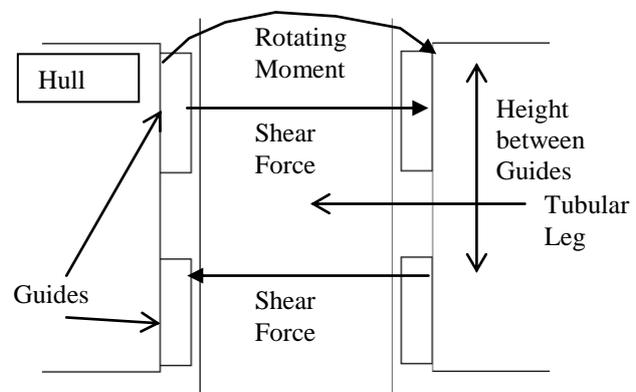


Figure 2 - Detailed Leg to Hull Connection

The key equation for rotational stiffness (k) is the rotating moment (M) over the resultant rotation (θ) as shown in Equation 1.

$$k = \frac{M}{\theta} \quad \text{(Equation 1)}$$

The environmental loading causes the hull to move away from the loading, which causes the parts of legs below the guides to bend. However, the part of the leg held in between the guides is unable to bend and so it shears. The shear stress (τ) felt by the leg is equal to the shear force (SF) over the cross sectional area (A) of the leg. Shear force is equal to the moment divided by guide spacing (H) as shown in Equation 2.

$$\tau = \frac{SF}{A} = \frac{M}{H \times A} \quad \text{(Equation 2)}$$

Because the shear force only acts on the half of the leg that is exposed to it, Equation 2 is divided by two to get the effective shear stress (τ_{eff}) in Equation 3.

$$\tau_{eff} = \frac{\frac{M}{H \times A}}{2} = \frac{2 \times M}{H \times A} \quad \text{(Equation 3)}$$

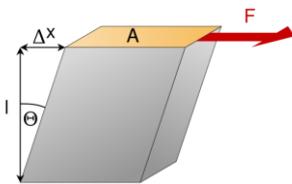
In order to find the angle at which the tube shears, the shear strain (δ) must be found and the equation for Shear Modulus (G) can be used to relate shear strain and shear stress.

The Shear Modulus is an unknown variable, but Young's Modulus (E) and the Poisson's Ratio (ν) of the steel used in the construction of the leg are known, which can be related to the Shear Modulus using Equation 4.

$$G = \frac{\tau_{eff}}{\delta} = \frac{E}{2(1+\nu)} \quad \text{(Equation 4)}$$

The equation is rearranged to find the shear strain in Equation 5.

$$\delta = \frac{2(1+\nu)}{E} \times \tau_{eff} \quad \text{(Equation 5)}$$



When a rectangle (a tube from a 2d perspective will look like a rectangle) shears to become a parallelogram, the shear strain is the change in length (Δx) divided by the height (H). Since the guides are resisting the shearing action, the angle (θ) will be very small and so that θ approximately equals $\tan \theta$.

$$\theta \approx \tan \theta \quad \delta = \frac{\Delta x}{H} = \tan \theta \Rightarrow \delta \approx \theta \quad \text{(Equation 6)}$$

Now that θ has been found to approximately equal shear strain for small θ in Equation 6, Equation 1 is substituted into Equation 5 and the rotational leg to hull stiffness (k) is found in Equation 7.

$$\theta \approx \frac{2(1+\nu)}{E} \times \tau_{eff} \quad k = \frac{M}{\theta} = \frac{M}{\frac{2(1+\nu)}{E} \times \tau_{eff}} = M \times \frac{E}{2(1+\nu)} \times \frac{1}{\tau_{eff}} \quad \text{(Equation 7)}$$

Using Equation 3, effective shear stress is substituted in and the moment is cancelled out of the equation as shown in Equation 8.

$$k = M \times \frac{E}{2(1+\nu)} \times \frac{H \times A}{2 \times M} = \frac{E \times H \times A}{4(1+\nu)} \quad \text{(Equation 8)}$$

Key Information on Leg and Jack Frame for a Generic Rig

- Tube Wall Thickness = 0.04m
- Tube Radius = 1.00m
- \therefore Tube Cross Sectional Area (A) = $1^2 \pi - 0.96^2 \pi = 0.246 \text{ m}^2$ (3dp)
- Height between Guide Plates (H) = 5.00m
- Young's Modulus of Steel (E) = 210×10^9 MPa
- Poisson's Ratio (ν) = 0.3

Therefore, rotational stiffness of the tubular leg to the guides in the jacking frame can be estimated as shown in Equation 9.

$$k = \frac{E \times H \times A}{4(1+\nu)} = \frac{210 \times 10^9 \times 5 \times 0.246}{4(1+0.3)} = 4.9 \times 10^{10} \text{ Nm/rad} \quad \text{(Equation 9)}$$

This can then be made more accurate using a Finite Element model of the leg to hull connection, but it is good enough for a first pass approximation.