

Mathematics and multiple-unit awards

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Executive summary

- **A proposal has been made by the awarding bodies that AS and A Levels in Mathematics and Further should be replaced by 3-unit, 6-unit, 9-unit and 12-unit awards.**
- **The consequences of the proposal have clearly not been thought through.**
- **This proposal was made on an earlier occasion, in 2003/4.**
- **Analysis at that time showed it to be deeply problematic.**
- **It would discourage students from fulfilling their potential, and so would reduce standards of attainment in sixth form mathematics.**
- **It would inflict particularly severe damage on Further Mathematics.**
- **Uptake for the 9-unit and 12-unit awards would be much less than that for AS and A Level Further Mathematics and might well become vanishingly small.**
- **All the highly successful work in promoting Further Mathematics in recent years would be written off.**
- **The qualifications would be more complicated than those at present for the awarding bodies to administer.**
- **The qualifications would not be transparent to end-users.**
- **It would be hard to make the system fair to students.**
- **Although the proposal is obviously well intended, it nonetheless seems like a death wish on mathematics at this level and consequently is in conflict with national interests and also the direction of government policies.**

1. Introduction

This paper is MEI's response to a proposal to replace the existing system of grading A Level Mathematics and Further Mathematics with one based on 3-, 6-, 9- and 12-unit awards, contained in the paper *Grading GCE Mathematics and Applied GCE Subjects*¹ written for the awarding bodies in April 2007.

The idea is not new. It was raised during the last round of syllabus changes in 2003/4 and was rejected then. Given the cogency of the arguments that were brought to bear against the proposal at that time², it is disturbing that it should be re-considered at all, let alone so soon.

There are two different types of argument against such a scheme.

Mathematical concerns

The proposed change would reduce the uptake of sixth form mathematics and, particularly, Further Mathematics in our schools and colleges. Consequently its introduction would be accompanied by a reduction in standards of attainment. The reasons for this are explained in the next section of this paper.

Technical grading and reporting issues

The proposed new scheme would inevitably result in a grading system that would be more complicated and harder for everyone to understand and work with than the present system. The reasons for this are described in some detail in Sections 3, 4 and 5. These sections also include comments on the three reasons for re-considering the proposal that are given in the paper *Grading GCE Mathematics and Applied GCE Subjects*.

- The introduction of the “no-decline” rule from January 2008³.
- The new practice of reporting unit grades (in addition to qualification grades) to universities.
- The fact that a number of overseas universities and some Cambridge admissions tutors have started to ask for a Uniform Mark total in A Level Mathematics that is above the threshold of 480 (out of a possible 600) for grade A.

In considering the various issues, it is important to understand the national need for more students to be taking more mathematics. That has to be paramount. Some of the issues involved are commented on in more detail in a paper by Peter Thomas⁴.

While grading and reporting need to be done as efficiently and transparently as possible, these are no more than stages within our overall provision and must not be allowed to dominate it. It is important that these processes are never seen as more than part of a much bigger picture.

2. Mathematics uptake

The proposed change would reduce the uptake of Further Mathematics, quite possibly to zero.

A common situation at the moment is that a student obtains a higher grade on A Level Mathematics than on Further Mathematics, for example A on Mathematics and C on Further Mathematics. A difference of 1 or 2 grades is very common. Such a student can present grades A and C with some pride to many universities. They carry the message “I have a top grade on the mathematics that everyone does but I have also studied a lot more mathematics and with reasonable success”.

However, if the proposal were to be accepted, such a student would be given a 12-unit award at BB. The effect of taking Further Mathematics would be to reduce the Mathematics grade from A to B. This is not some obscure case; it is what would happen to large numbers, probably the majority, of Further Mathematics students.

Consequently, all students opting for Further Mathematics would knowingly be setting their grades in Mathematics at risk. It is hard to imagine a more powerful disincentive from doing Further Mathematics. Few, if any, students would do it. Further Mathematics would be killed off at the stroke of a pen. All the highly successful work, particularly from the Further Mathematics Network⁵, that has recently produced a strong resurgence in Further Mathematics would be thrown away.

Those who currently take Further Mathematics gain a much higher level of mathematical fluency and this raises the quality of their work in the Mathematics A Level as well. So destroying Further Mathematics would also have the effect of lowering standards in the remaining A Level.

It is predictable that much the same effects would be observed between AS and A Level Mathematics. Going on from a 3-unit award (i.e. AS Level) to a 6-unit award (A Level) would set the AS grade at risk and so more students would choose to drop mathematics after AS Level, rather than go on to the full A Level in Year 13.

Thus this proposal would not only write off Further Mathematics but also have a detrimental effect on A Level Mathematics. It is hard to believe that it could have been put forward at a time when government money is being put into programmes to raise the profile of sixth form mathematics, for example the Further Mathematics Network, the STEM initiative⁶ and More Maths Grads⁷.

3. The present situation

This section describes current practices. With some modifications, these have been in place since 1992 and so have been successfully used to grade very large numbers of candidates.

Mathematics differs from other subjects in having two distinct but interlocking A Levels, Mathematics and Further Mathematics. The description below starts with the case of a candidate taking the 12 units needed for these two A Levels.

The marks given in the examples are Uniform Marks and the grade A threshold is 480 out of a possible 600.

Grading 12 units

The basic question is how best to allocate 6 out of a candidate's 12 units to A Level Mathematics and the other 6 to A Level Further Mathematics. Grading other possible numbers of units, like 9 and 13, involves using the same principles. Reading *Grading GCE Mathematics and Applied GCE*, one would be tempted to imagine that this is very complicated. This is not the case.

Grading was made much simpler with the 2004 syllabus revisions when the number of possible combinations of units for A Level Mathematics was reduced to 6. All candidates take C1, C2, C3 and C4. The other two units must be one of the following pairs: M1 & M2; S1 & S2; D1 & D2; M1 & S1; M1 & D1; M1 & S1. No other combinations may be used for A Level Mathematics⁸.

Those units that are not used for A Level Mathematics count instead towards Further Mathematics.

To appreciate the situation, consider the possible outcomes for a candidate taking the following 12 units and obtaining the Uniform Marks indicated.

| | | | | | | | | | | | |
|----|----|----|----|-----|-----|-----|----|----|----|----|----|
| C1 | C2 | C3 | C4 | FP1 | FP2 | FP3 | M1 | D1 | S1 | S2 | S3 |
| 87 | 86 | 81 | 81 | 85 | 80 | 80 | 75 | 60 | 85 | 80 | 85 |

There are just 4 ways that the grading could be carried out.

| Mathematics | | | Further Mathematics | | |
|---------------|-------|-------|---------------------|-------|-------|
| Units | Total | Grade | Units | Total | Grade |
| C1-4, S1 & S2 | 500 | A | FP1-3, M1, D1 & S3 | 475 | B |
| C1-4, S1 & M1 | 495 | A | FP1-3, D1, S2 & S3 | 480 | A |
| C1-4, S1 & D1 | 480 | A | FP1-3, M1, S2 & S3 | 495 | A |
| C1-4, M1 & D1 | 475 | B | FP1-3, S1, S2 & S3 | 500 | A |

Under the present procedures, the first decision is what is the best grade available to the candidate for A Level Mathematics, and in this case it is A. There are three ways that this can be done, and the fourth possibility, which gives grade B for Mathematics is ruled out. This process of selecting the best grade for Mathematics is the “best” part of the least-best rule; it is built into the systems used by the awarding bodies.

Three possibilities are now left.

| Mathematics | | | Further Mathematics | | |
|---------------|-----|---|---------------------|-----|---|
| C1-4, S1 & S2 | 500 | A | FP1-3, M1 D1, & S3 | 475 | B |
| C1-4, S1 & M1 | 495 | A | FP1-3, D1, S2 & S3 | 480 | A |
| C1-4, S1 & D1 | 480 | A | FP1-3, M1, S2 & S3 | 495 | A |

The second decision is which of the remaining combinations of units should be selected. This is determined by the “least” part of the least-best rule. In this case, the three possible totals for A Level Mathematics are 500, 495 and 480 and the unit allocation which gives the least of these, 480, is the one selected; it is shaded in the table.

The least-best rule is designed to pass the maximum available credit on to the Further Mathematics, and so ensure that the best Further Mathematics grade is also awarded.

If, by contrast, a best-best rule were applied, the combination selected would be that which gives the candidate A in Mathematics and B in Further Mathematics. Such a rule would clearly lower Further Mathematics grades and so make it a less attractive subject. In consequence fewer students would take Further Mathematics; this is manifestly not a course of action that should be pursued.

The present system works well. It is designed to give candidates the best qualification grades available to them and it does so efficiently.

However, it can be undermined if end-users, particularly admissions tutors start asking for marks rather than grades. In the example above, there are two ways that the candidate could be awarded grade A in both Mathematics and Further Mathematics. In one, that currently awarded, the marks are 480 and 495 respectively, in the other they are 495 and 480.

The procedure could be extended by introducing an extra optimisation decision before the final allocation of units is made, so as to give the maximum possible total Uniform Mark to Mathematics within the constraint of obtaining the best pair of grades. While this is technically possible, it would involve some cost to the awarding bodies in terms of programmers' time.

Alternative and better strategies are available to admissions tutors:

- to consider the results of all the units (in this example 12) that students have taken rather than just the 6 that are allocated to A Level Mathematics;
- to ask for a Further Mathematics qualification rather than a total above 480 on A Level Mathematics.

In *Grading GCE Mathematics and Applied GCE subjects* the fact that some admissions tutors are now asking for totals of more than 480 is presented as a major problem. It is not. It applies mainly to overseas universities as well as a few Cambridge colleges. We would be quite wrong to let our national system be subverted from outside the country or by a handful of Cambridge admissions tutors who would no doubt change their practice if they fully understood the grading process.

Grading 9 units

The procedure for the award of 9 units, is the same as that for 12 units apart from the obvious difference that, as well as A Level Mathematics, candidates obtain AS rather than A Level in Further Mathematics.

Extra units

It is not uncommon for a student to take an extra unit. The table below shows the results of a candidate who is taking A Level Mathematics but has done 7 units, one more than is necessary.

| | | | | | | |
|----|----|----|----|----|----|----|
| C1 | C2 | C3 | C4 | M1 | D1 | S1 |
| 87 | 86 | 81 | 81 | 85 | 60 | 90 |

There are 3 ways that this student can be awarded A Level Mathematics, all of them at grade A.

| Mathematics | | |
|--------------------|--------------|--------------|
| Units | Total | Grade |
| C1-4, S1 & M1 | 510 | A |
| C1-4, S1 & D1 | 485 | A |
| C1-4, M1 & D1 | 480 | A |

At present the awarding bodies' computers are programmed to apply the least-best rule in this situation so that M1 and D1 are allocated to the A Level, and S1 is left spare. This makes the high scoring S1 unit available in case the candidate should want to go on to Further Mathematics (something the awarding body cannot know about) but it leads to the criticism that the candidate's best unit is the one that is not counted.

Providing that admissions tutors understand the grading process and look at an applicant's complete set of results, this need not be a problem. However, the situation could be improved by the introduction of an optimisation decision (as already described) immediately before the allocation is made

Early certification

The situation described for extra units arises mainly because some candidates apply for certification early, for example at the end of Year 12 instead of waiting to the end of Year 13. The whole grading procedure is much simpler if, as in the first example, it is left until the very end.

A more common case of early certification occurs when a student certifies AS Level at the end of Year 12 and then continues the subject to A Level in Year 13. The AS certification is quite meaningless in that case and should be discouraged or disallowed. If the student continues to A Level it will be overwritten and if the student gives up mathematics it is still available to be claimed in Year 13 (it cannot be lost).

When problems arise and students could possibly be disadvantaged, they are almost always caused by early certification. (The reason for this will become clear in the next section.) It should be seen as good practice for schools and colleges to discourage the practice.

Current UCAS requirements are unhelpful in this, being interpreted by many students as being close to an instruction to certify early.

You must provide full details for the following qualifications to help universities and colleges consider your application. These include: all qualifications for which you have accepted certification from an awarding body, even if you are retaking all or part of the qualification; and if you have certificated the result for a GCE or VCE award, you may enter details for the units that make up this award. You can enter unit details for certificated GCE

AS, GCE AS Double Award, GCE A level, GCE A level Double Award, Advanced VCE and Advanced VCE: Double Award qualifications. You cannot enter details for units that make up uncertificated GCE or VCE awards. Information about performance or achievement in these units can be included in your personal statement or reference.”⁹

It is clearly important that the information requested on UCAS forms reports on applicants' progress without interfering with it. An alternative approach would be for provisional certifications to be issued at the end of Year 12, stating what grades students would receive if they were to certificate at that time.

A different group of students are those who certificate in good faith at the end of Year 13 but then return to school or college to re-sit the whole year. Although there are not many of these students, it must be possible to deal with them fairly.

4. The “no-decline rule”

Although the present procedures work reasonably well, a change is to be introduced in January 2008 which will create difficulties. Imagine a student who takes the units during Year 12 and uses them to certificate A Level Mathematics.

| C1 | C2 | C3 | C4 | M1 | S1 | Total | Grade |
|----|----|----|----|----|----|-------|-------|
| 87 | 86 | 81 | 81 | 85 | 30 | 450 | B |

All this student's units are grade A (80+) apart from S1 which pulls the overall result down to an A Level grade B. Naturally the student wants to re-sit and obtain grade A.

At present the student is entitled to decline the A Level certification. Once this is done, all the units are freely available for a new certification. It is as if no certification had ever taken place. This particular student has clearly not got on nearly as well in Statistics as in Mechanics and so could decide that M2 would be a better option than re-sitting S1, leaving the S1 as a spare unit. This would be entirely legal.

However, when the no-decline rule comes in the student will no longer be allowed to decline the A Level. It will be possible to re-sit the units within it, so that a re-sit of S1 will be allowable and it will still be possible to replace it with M2. However, the S1 will no longer be a spare unit; it will still be locked into the Mathematics A Level.

If the student does no more mathematics this will not matter. Problems will, however, arise if the student goes on to take Further Mathematics in Year 13.

Imagine that the same student, as well as re-sitting S1, takes 6 more units, for A Level Further Mathematics.

| | | | | | | |
|----|-----|-----|-----|----|----|----|
| S1 | FP1 | FP2 | FP3 | M2 | M3 | M4 |
| 59 | 90 | 86 | 84 | 90 | 90 | 80 |

The student's possible certifications would be as follows. (Notice that with these units there are only 2 ways that A Level Mathematics can be awarded.)

| Mathematics | | | Further Mathematics | | |
|---------------------|-----|---|---------------------|----------|---|
| C1-4, M1 & S1 | 479 | B | FP1-3, M2, M3 & M4 | 520 | A |
| C1-4, (S1), M1 & M2 | 510 | A | FP1-3, M3 & M4 | No award | |

Thus the student can either accept a grade B in Mathematics and obtain an A in Further Mathematics, or obtain a grade A in Mathematics and be ungraded in Further Mathematics because only 5 units are available.

By contrast a different student who had done exactly the same units at the same time but had not certificated Mathematics at the end of Year 12 would be awarded grades A in both Mathematics and Further Mathematics.

| Mathematics | | | Further Mathematics | | |
|---------------|-----|---|---------------------|-----|---|
| C1-4, M1 & M2 | 510 | A | FP1-3, S1, M3 & M4 | 489 | A |

That situation is manifestly unsatisfactory.

However, it can easily be dealt with by unlocking any units that are left over from the Mathematics A Level (S1 in this example) once the grading of Mathematics has been completed. This procedure would produce the same outcomes as that currently in place and so would involve the awarding bodies in no additional programming costs.

5. The proposal for 3, 6, 9 and 12 unit awards

The proposal in *Grading GCE Mathematics and Applied GCE Subjects* is contained in the following recommendation.

Award 'single' qualifications in GCE Mathematics and in the Applied GCE subjects. These 'single' qualifications will be awarded at the following thresholds: for 3 unit results, 6 unit results, 9 unit results and 12 unit results (for GCE Mathematics, these qualifications will be extended to 15 unit results and 18 unit results.

The paper describes this as a “straightforward” solution. This is misleading. The description given is lacking in detail and, as was discovered in 2003/4, it is only when the detail is considered that the full complexity becomes apparent. However, the two examples that follow indicate some of the problems lurking beneath the surface.

Example 1

Two candidates applying for 6-unit awards.

One has done 6 units that would currently be associated with A Level Mathematics.

| AS units | | | A2 units | | |
|----------|----|----|----------|----|----|
| C1 | C2 | S1 | C3 | C4 | S2 |

The other has taken 3 AS Mathematics units and 3 more AS units for AS Further Mathematics.

| AS Mathematics | | | AS Further Maths | | |
|----------------|----|----|------------------|----|----|
| C1 | C2 | S1 | FP1 | M1 | D1 |

The two profiles are very different; the second candidate has covered none of the A2 Subject Core for Mathematics, something that end-users would certainly want to know.

When this situation was considered in 2003/4, it was decided that the only way round it was to have two different 6-unit awards: 6-unit (AS Level) and 6-unit (A Level).

The concept of a single 6-unit award is not sustainable.

Example 2

A student who does 6 A level units in Year 12 with the following results.

| C1 | C2 | C3 | C4 | S1 | S2 | Total | Grade |
|----|----|----|----|----|----|-------|-------|
| 87 | 86 | 81 | 81 | 85 | 40 | 460 | B |

The student obtains and certificates a grade B in the 6-unit (A Level) award at that stage, but then goes on to take 3 more units in Year 13 for AS Further Mathematics.

| | | |
|-----|----|----|
| FP1 | M1 | D1 |
| 45 | 40 | 60 |

What award does the student now obtain ?

- It is not entirely clear how 9-unit grading would work but it is likely that the 9-unit total of 605, would probably correspond to CC. If so, the student's A Level grade would have gone down from B to C as a result of taking the extra units.
- The 6-unit (A Level) total can be improved to 480 if S2 is replaced by D1. This would give a grade A but only if the student is allowed to throw away the remaining 3 units, and only if the awarding body is able to handle such a situation.

By contrast under the present arrangements this student would be awarded A in A Level Mathematics and E in AS Further Mathematics.

- Another possibility is for the student to obtain a 6-unit (AS Level) award. This would be based on C1, C2, S1, FP1, M1 and D1. With these figures, this would be an unattractive option, giving a grade C on 403, but no doubt situations would arise where it was a sensible award to request.

Thus, far from producing a simple scheme, the proposal will produce one that is much more complicated than that at present in place. The no-decline rule had not been invented in 2003/4 but it will certainly make the situation even more complicated.

The system should both encourage students and be fair to them and the existing scheme achieves both. The two examples above show something of the difficulty of achieving either with the proposed new scheme.

6 In conclusion

The awarding bodies' intention in proposing these changes to the grading system is that they will make processing easier. No one is suggesting that they will improve the amount or quality of mathematics that students learn. It is, however, clear that if they were adopted, many students would learn less mathematics and that overall standards of attainment would fall significantly.

It is hard to avoid seeing the proposal as a death wish on mathematics in this country.

References and Notes

1. Grading GCE Mathematics and Applied GCE subjects, J.Pritchard, 2007.
2. The awarding of AS and A Level Mathematics and Further Mathematics: *The proposal for 3-, 6-, 9- and 12-unit awards*, MEI, 2004 (available at www.mei.org.uk)
3. Notice to Centres: A change to GCE, Applied GCE and GNVQ certification and decline rules. JCQ, September 2006
4. Comment on *Grading GCE Mathematics and Applied GCE subjects*, Peter Thomas, April 2007.
5. See wwwfmnetwork.org.uk
6. See www.dfes.gov.uk/stem/
7. See wwwmoremathsgrads.org.uk
8. The MEI specification has two different second level units in Decision Mathematics, D2 and DC (Decision Mathematics Computation) and so in this case a 7th A Level combination is available.
9. UCAS online, 2007 entry